

FORGOTTEN TECHNIQUES AND GEOMETRIC RULES OF DIAMOND VAULT CONSTRUCTION REDISCOVERED IN VIRTUAL SPACE

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Abstract

The research problem of this project was to identify the principles governing the geometric structure of medieval diamond vaults. The Church of St. Peter and Paul in Gdańsk was the subject of detailed analysis. The research was based on a 17th-century old print by Barthel Ranisch, an analysis of available literature and contemporary inventory documentation, and in situ observations. A geometric interpretation of the vaults in the northeastern nave of the church was performed. A theoretical model of the vault was obtained, with a structure similar to the 16th-century original. The analogy between the virtual construction and the actual solution points out the correctness of the method used. The obtained results open up the possibility of conducting further research, the aim of which would be to confirm the application of the discussed geometric principles in the construction of other crystal vaults from this period.

Keywords: Gothic Crystal Vaults; Virtual Vault Model; Barthel Ranisch; Church of St. Peter and St. Paul in Gdańsk

Introduction

Diamond vaults are brick structures, a type of late Gothic roofing characterized by vault cells in the form of pyramidal or concave recesses. In early designs, they were combined with ribs. From the late 14th century, cross-ribbed vaults with concave ridge lines and convex brick bowls were built in the Lower Rhine, the Netherlands, and Westphalia [1]. However, most often, these are forms with sharp ridges—without ribs. Covered with plaster, they create interesting visual effects thanks to contrasts of light and shadow. In many buildings, they can be seen in other types of Gothic vaults, especially the ribbed vaults (with rectilinear vault cells spanning between ribs), popular throughout the Gothic period. An example of this type of building is the Church of St. Peter and St. Paul discussed in this paper. The ribbed vaults can be found in the main nave, presbytery, and sacristy, whereas diamond vaults are in the side aisles.

Gothic diamond vaults are rare. They can be seen in several European countries: Austria, Belarus, the Czech Republic, Lithuania, Latvia, Germany, Poland, Russia, and Slovakia (Milada and Oldrich Rada, in their work *Das Buch von den Zellengewölben*, collected and discussed all Gothic diamond vaults found in European countries [1]). They are found in churches, monasteries, castles, residential buildings, and town halls. Originating in Germany (Albrechtsburg in Meissen), they were developed in Europe at the turn of the 15th and 16th centuries [2]. Although they were built in Gothic for a relatively short period, in later periods they inspired builders to incorporate similar forms into their interiors. In St. Hyacinth's Church in Warsaw, the nave was vaulted with diamond vaults in the 17th century. At the end of the 19th

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century, during the Gothic Revival period, vaults reminiscent of a bygone era appeared. The diamond vaults at the Collegium Novum in Kraków, for example, date from this period. Another example of Gothic-inspired construction is the vault at the Church of the Blessed Karolina Kózkówna in Tarnów, built at the end of the 20th century [3].

Compared to the abundant scholarly literature on Gothic ribbed vaults, crystal structures are a rarely discussed and not very well-researched topic. Even more challenging is finding information on their spatial structure. These are structures of beautiful and visually stunning yet exceptionally complex construction. Their construction required a perfect mastery of the principles of spatial geometry. The art and technique of building these vaults five centuries ago have been almost completely forgotten. At the same time, for conservation reasons related to monitoring the technical condition of buildings and carrying out necessary repair and reconstruction works, there is a constant need to expand this knowledge [4], [5], [6]. The topic discussed in this paper addresses this need.

In European countries, the greatest damage to historic buildings occurred as a result of operations related to World War II. Poland suffered enormous losses at that time. Some buildings were destroyed and never rebuilt. Others were left as permanent, protected ruins. Repairs and reconstructions of damaged monuments continued for many years after the war. Gdańsk was largely destroyed as a result of the Soviet offensive, and the reconstruction of some historical monuments was not completed until the 21st century. Among them was the Church of St. Peter and St. Paul, which was restored from ruins in 2006 [7].

One of the most important aspects of rebuilding damaged Gothic vaults is knowledge of the geometric principles governing their spatial structure. This knowledge allows for the correct reconstruction of the vault's shape and the appropriate integration of reconstructed elements with the preserved remnants of the original vault. This type of problem arose, for instance, while reconstructing the vault in the castle church in Malbork (completed in 2016) or during the vault reconstruction in the tower of St. Mary's Church in Gdańsk (completed in 2018) [8].

The principles governing vault geometry have been discussed by many researchers, usually in relation to a given pattern and its location. For example, Spanish researchers discuss specific types of vaults, such as asymmetric [9], sexpartite [10], and vaults with curved liernes [11]. Their research relies on 16th- and 17th-century sources, including a 16th-century manuscript by Rodrigo Gil de Hontañón [11], [12]. In Poland, the oldest work on the subject of Gothic vaults is the 17th-century work by Barthel Ranisch called *Beschreibung aller Kirchen-Gabaude der Stadt Dantzic* from 1695 [13]. This source is usually referred to in the context of research on the vaults of Gdańsk buildings [14], [15], [16].

The ongoing Russian-Ukrainian war in Europe and the resulting destruction of historic buildings, as well as other numerous disasters (earthquakes, fires, hurricanes, floods, etc.), urge us to expand and gather as much knowledge as possible about these extremely valuable relics of times gone by. Earthquakes in Italy are responsible for widespread destruction and remarkably significant loss of historic buildings. For example, as a result of the earthquake in 1997, the vaults in the Basilica of St. Francis of Assisi suffered significant damage [17], [18]. In 2019, the entire world witnessed as the Cathedral of Notre Dame in Paris, an icon of French Gothic architecture, kept burning and ultimately suffered extensive damage. In Poland, fires also affected historic buildings. Among the most unlucky ones is St. Catherine's Church in Gdańsk, which burned three times (1905, 1945, and 2006). The Church of St. Peter and St. Paul in Gdańsk, rebuilt in 2006, also burned in 2019. Therefore, there is an urgent need to document and collect information about historic buildings. The research project presented in this paper is tied in adequately with this type of effort. It focuses on deciphering the principles governing the geometric structure of medieval diamond vaults.

First, the work of Barthel Ranisch, who inventoried all Gothic churches in Gdańsk [13], was analyzed. He described, among other things, the method of constructing diamond vaults in the Church of St. Peter and St. Paul. This record formed the basis for subsequent research,

including a detailed analysis of the original diamond vaults located in the side aisle (northeast) of the church. Using contemporary inventory documentation (including a point cloud from the interior scan) and in-situ observations, an attempt was made to construct a virtual model of one of the five bays of the side aisle in question. The construction of the virtual structure involved the use of state-of-the-art modeling tools, including the algorithmic-parametric program Rhinoceros with the Grasshopper add-on. The use of such tools is currently becoming increasingly popular, especially in collecting information about historical buildings within Historic (or Heritage) Building Information Modeling (HBIM) [19]-[26]. Applying the principles of the old masters, a theoretical model of the vault was obtained, with a structure similar to the one found inside the church.

Materials and Methods

The conducted study employed a historical-interpretive research method [27]. Materials concerning crystal vaults, particularly those related to the ones found in Gdańsk, were collected and analyzed. The research techniques employed at this stage included the analysis of literature, available documentation, and archival materials. The research (undertaken) included data search and assessment of its value as well as explanation and interpretation of the collected materials.

The primary research method employed in this project was a case study. The research techniques used in this method included:

- site visit, collection of photographic documentation and analysis of photographs,
- review of available literature and 17th-century construction drawings,
- analysis of the vault plan in relation to historical rules governing their design,
- creation of a computer 3D model visualizing the spatial structure of the vault.

The research techniques used in this study involved primarily computer modeling. The final result of these activities was digital models of the vault constructed according to the method described by B. Ranisch. Two programs were used for this purpose: AutoCAD, which was used to analyze the vault's plan, construct the ridge arches, and build a wireframe model. The Rhinoceros 3D algorithmic-parametric program, along with the Grasshopper add-on, was used to model the vault's cells. For comparison purposes, a point cloud survey conducted by Aurea Porta Conservation of Monuments was used.

Diamond Vaults – Characteristics

Ribless diamond vaults, unlike ribbed vaults, were built for a relatively short period—between 1471 and 1550 [15]. The origins of these structures have been debated by many Central European historians. This topic was developed by Clasen in his work *Deutsche Gewölbe der Spätgotik* (1958) [28], which was followed by studies by Milady and Oldrich Rad [1], [29]. In Poland, Maria Brykowska presented an article on this subject in 1965 [30]. According to historians, crystal vaults were not built before 1471 (the beginning of the construction of the Albrechtsburg in Meissen) or after the first half of the 16th century.

Diamond vaults originate from Gothic traditions, and their builders never abandoned the methods used for building ribbed vaults characteristic of their region. In its simplest form, they adapted the cross-ribbed vault to the crystal version, eliminating the ribs, which were replaced by ridges. More complex forms developed as vaults with star or net motifs, based on groin, barrel, or dome-shaped forms. Late Gothic diamond vaults are constructed exclusively of brick, with the exception of a few connected to stone ribs.

The construction technology for diamond vaults typically followed traditional methods used for ribbed vaults. First, a network of the longest arches was built on wooden centerings, then support for the shorter ridges was constructed (as Ranisch shows in the section on the diamond vault at St. Mary's Church) [13]. Sometimes they were also built without using supporting structures, which would explain why they appear uneven in some smaller churches. The space

between the ridges was then filled with sharply folded cells. Sometimes they were modeled differently, as rounded forms. The grid or shell of prismatic (or circular) cells was built without any supporting structure because it was too steep and unwieldy to install a wooden framework. Bricks were laid with their edges on the wooden centerings to direct the angle of the cell. To mask irregularities at the connecting points, especially in smaller churches, round discs were placed over them, and the entire vault was covered with a layer of lime.

The analysis of preserved diamond vaults shows that changes in their form followed the transformations of ribbed vaults from the same period. By the third decade of the 15th century, large surfaces with such diamond cells began to emerge—in both ribbed and ribless vaults, with star and net motifs. By the mid-16th century, however, they were smaller, more delicate, and discreet. Over time, as the entire vault surface became fragmented and increasingly flat, and the cells lost their prismatic form, the effect of light and shadow disappeared. The cells became increasingly less visible and less plastic, until finally, after the mid-16th century, they disappeared completely.

Known examples of ribless vaults do not exceed 7.5 meters in span [30]. In Gdańsk churches, this type of vault was built only in the side aisles. It appeared exceptionally in the (main) nave (with a span of 7 meters) and side aisles in a small church in Kętrzyn.

Gdańsk Diamond Vaults

The exact date of the creation of diamond vaults in Gdańsk is unknown. Early solutions, such as the Saxon ones in Meissen, consist of segments with large surfaces, deeply carved, and sharply angled. Later designs (around the year 1500) are characterized by finer, shallower, and smaller segments. Taking this as a pattern for the development of crystal vaults in Gdańsk, it can be seen that the vaults with the largest cell areas were built in the Franciscan monastery (ca. 1496). Franciscan builders probably brought this technology from Saxony, introducing it throughout Prussia. Diamond vaults were constructed in several interiors of the monastery: the refectory, the cloisters, and the kitchen (Fig. 1a). In the refectory, diamond fans supported along the walls seamlessly intertwine with the fan fields, which converge around the central pillar. The interior is intimate, and as it is relatively low, the fields of the crystal segments are clearly visible. The sculptural effect is expressive and dynamic. Sharp contrasts of light and shade, variable sizes and inclinations of fields, and an abundance of multidirectional lines and arches create a stunning aesthetic impression, having a stronger impact on the viewer than in the high vaults of the interiors of Gdańsk churches. The cloisters feature interconnected rows of eight-pointed stars. These patterns, often surrounded by net formations, subsequently developed in Gdańsk and almost all of Prussia. The diamond vaults in St. Mary's Church are among the earliest in Gdańsk (Fig. 1b). According to the agreement of February 28, 1501, on that date, Master Heinrich Hetzel was hired to vault both side aisles of the nave. The diamond vaults were constructed in the southern and northern aisles, forming low domes. In this church, the side aisles are separated from the nave by wide transverse arches (arcades), which distinguishes them from Czech structures. The cells of individual bay fields form eight-pointed stars at the top. The stars do not extend across entire bays but are surrounded by numerous diamond-shaped lattice cells that connect the stars to the bay boundaries and to each other. Thin, transverse arches separating the bays are preserved.

Eight-pointed stars combined with net forms, such as those found in St. Mary's Church, developed from 1500 in Prussia, reaching Lithuania and Estonia. Probably around the same time (ca. 1502), diamond vaults were built in St. Catherine's Church in the side aisles, under the tower, and in two chapels (Fig. 2a). The sharply cut and relatively small (ca. 5 m wide) eight-pointed stars create a striking visual effect, accentuated by the church's low height (13 m). Similar diamond vaults can be found in St. Peter and St. Paul Church (Fig. 3). The motif is an eight-pointed star spread out at the top of a low dome, although here the vault has no diagonal or longitudinal arches. The star and net diamond vaults in the eastern and western choirs of St. Bridget's Church were built in the early 16th century (Fig. 2b). The vaults of the western choir

remain unchanged to this day. Diamond vault cells extend directly from the wall, creating eight-pointed star patterns at the top of the dome. In St. Barbara's Church, diamond vaults were constructed under the tower and in the sacristy during the church's renovation in the early 16th century. Unfortunately, they collapsed as a result of a fire in 1945. In the Church of St. John, the diamond vaults were constructed only in the Library of Zappio. According to Dehio, the vault was constructed before the 16th century, although small cells may indicate a later date. In the Church of St. Bartholomew (1500), the diamond vaults with deeply cut, irregular segments were constructed in the sacristy.

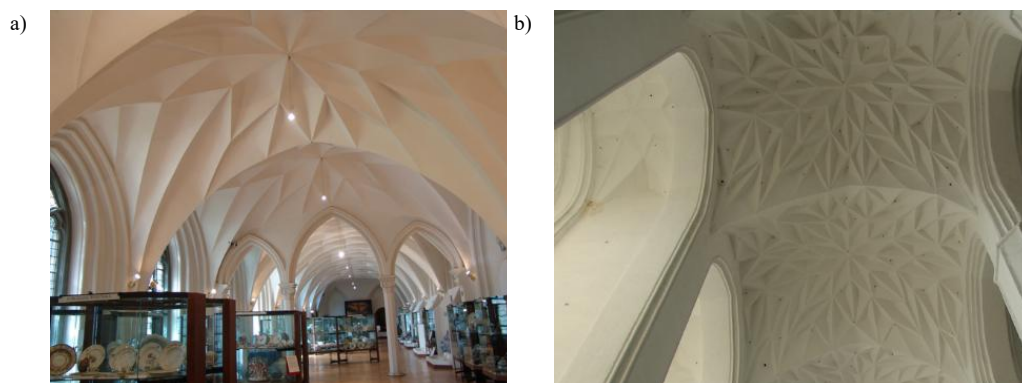


Fig. 1. Diamond vaults in Gdańsk: a) National Museum (former Franciscan monastery); b) St. Mary's Basilica

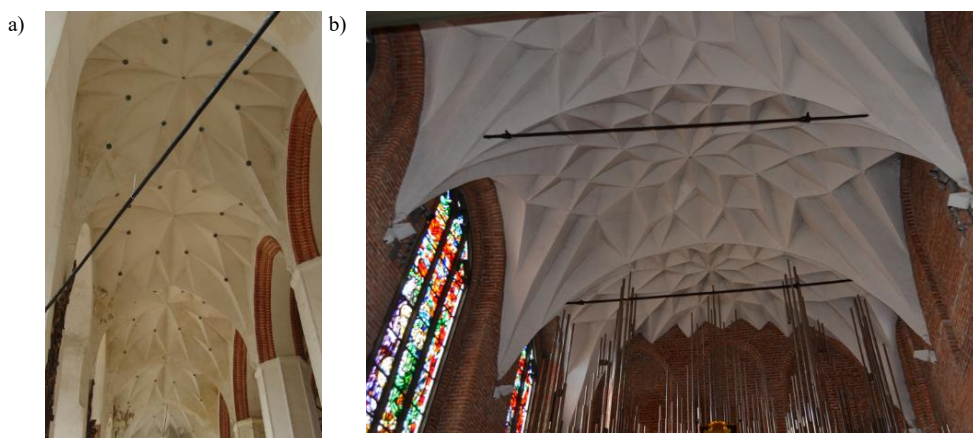


Fig. 2. Crystal vaults in Gdańsk: a) St. Catherine's Church; b) St. Bridget's Church

In conclusion, it has to be acknowledged that the construction of diamond vaults in almost all of Gdańsk's churches gave the city's architecture a special distinction and charm. Even today, six churches still have original diamond vaults: St. Mary's Basilica, the Church of St. Catherine, St. John, St. Bridget, St. Peter and St. Paul, and the Franciscan Monastery (now the National Museum). The chiaroscuro effects of the crystal forms were achieved through the sculptural modeling of deeply carved surfaces and the contrasting combination of surfaces joined at varying, often acute, angles (sharp cuts, as in crystals) and the covering of white plaster, which accentuates the visual effects. The Gothic churches of Gdańsk, which still display authentic crystal vaults, share several common structural features. They have three-nave hall layouts with rows of simple, octagonal, undecorated pillars and dome-like vaulting bowls, and most often they feature eight-

pointed star patterns in their finials. In general, the patterns of crystal vaults echo earlier ribbed patterns. The transverse arches remained continuous, though thin. The naves of churches are always separated by wide, transverse arcades. The division of individual bays is sometimes unclear due to the fragmentation of the abutting cells, but the transverse groins are always defined and uninterrupted. The oblique and horizontal groin almost always replaces the earlier oblique ribs. The long history of church building in Gdańsk makes it an interesting field of study for historians and architects. The pioneering work and research should be attributed to Barthel Ranisch. He is the one to be credited with describing, for the first time in history, the construction of vaults, including the precise procedures according to which the Gdańsk vaults were constructed. He shed light on the history and construction techniques of Gdańsk's buildings and published a pioneering, invaluable description of the general construction method for vaults in the late Gothic period.

Church of St. Peter and St. Paul in Gdańsk

The Church of St. Peter and St. Paul, located on Żabi Kruk Street in Gdańsk, is the parish church of the Stare Przedmieście quarter of the city. Construction began at the end of the 14th century. In 1424, the nearly completed church was destroyed by fire. Reconstruction began immediately. In 1456, when the church gained the status of the parish church of the Stare Przedmieście, work started to raise the height of the church, first the tower, and then the naves [7]. Only the chancel remained unchanged in height. The raised naves were then vaulted with net and diamond vaults (1514-1516).

The church served successively as a Catholic, Lutheran, Calvinist, apostolic Catholic, and Armenian Catholic church. In 1557, during the Reformation, the church was taken over by the Lutherans, and then in 1590, it was transferred to the Calvinists, and it remained their main place of worship until 1945. This influenced changes in the interior design as it was adapted to the needs of a particular religious group. The church was not spared the turmoil of war. The greatest damage occurred in 1945, when it was bombed and later set on fire during a Soviet assault, which resulted in the collapse of the tower and the vaults of the chancel. In the postwar period, the building, taken over by the communist authorities, continued to deteriorate. As a result of the weakening of the structure, some pillars and most of the vaults of the main and southern naves collapsed (in 1946 and 1947) [31]. In 1960, the first technical and operational project for the reconstruction was developed [7]. However, the attitude of the communist authorities in Poland to the Catholic Church and religion in general was rather hostile. This resulted in the blocking of reconstruction and repair efforts. Reconstruction was hampered by administrative methods, break-ins, and the destruction of sacral fittings and furnishings [7]. Construction work began only in 1975, and the reconstruction of the vaults was carried out in the years 1996-2006.

The Church of St. Peter and St. Paul is a three-nave structure built on a rectangular plan (Fig. 3c). It is 63 meters long, 23 meters wide, and 18 meters high. The entire interior of the church is vaulted. The five-bay naves feature two types of vaults: net (central nave) and diamond (side aisles) [7]. A common motif throughout the church is an eight-pointed star. The single-nave chancel and sacristy are covered with star-ribbed vaults (with a six-pointed star design). As for the external appearance of the church, what attracts attention is a fairly unusual, massive tower covered with a gable roof (Fig. 3a).

The church is clasped by external buttresses, between which are large, pointed-arch windows. The decorative gable of the porch with blind windows and richly profiled pinnacles contrasts with the austerity of the façade. The original diamond vault in the Church of St. Peter and St. Paul in Gdańsk is preserved on the northeastern side aisle (Fig. 3). Therefore, studies were conducted on these vaults in order to determine their geometric structure.

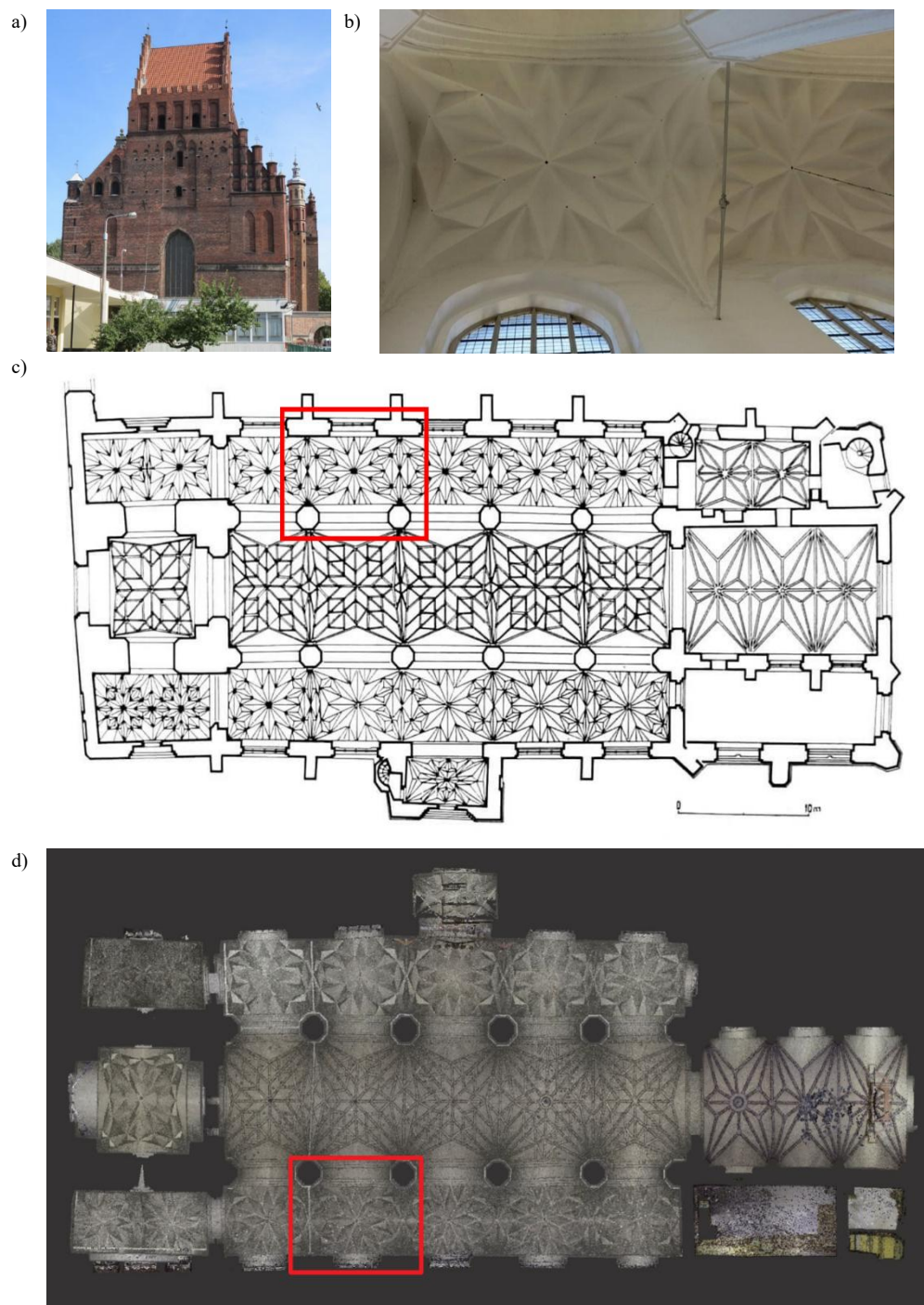


Fig. 3. Church of St. Peter and St. Paul in Gdańsk: a) western elevation; b) diamond vaults in the northeastern nave [source: a), b) <https://zabytek.pl/pl/obiekty/gdansk-kosciol-par-pw-sw-piotra-i-pawla>]; c) church plan with the analyzed bay shown in red square [source: <https://medievalheritage.eu/pl/strona-glowna/zabytki/polska/gdansk-kosciol-sw-piotra-i-pawla/>]; d) scan of the vault with the analyzed bay shown in red square [source: documentation prepared by Aurea Porta Monument Conservation Tomasz Korzeniowski]

Results and discussion

First, a detailed analysis of the 2D drawing of the vault was performed. In the next stage, a theoretical wireframe model. This structure formed the basis for the creation of a virtual version of the vault, incorporating pyramidal cells. At each stage of the modeling process, the obtained results were verified using archival, photographic, and numerical documentation in the form of a point cloud. This procedure enabled the construction of a model that was as close to the actual original solution as possible.

Analysis of the Vault Plan

The construction of the spatial model of the diamond vault in the Church of St. Peter and St. Paul was based on the construction records presented by B. Ranisch [13] (Fig. 4). In his work the drawings are not assigned to any numbered pages. They are most often attached to the chapter describing a given monument. Therefore, the drawings of the diamond vaults in the Church of St. Peter and St. Paul in Gdańsk can be found in the chapter describing this church (pages 63 to 67). The fragment from B. Ranisch's work shown in Figure 4a presents the vault plan (left side) and diagrams of the arches between the pyramidal vault cells (right side). The diagrams were used to prepare the supporting scaffolding for the construction of the vault in the form of wooden centerings.

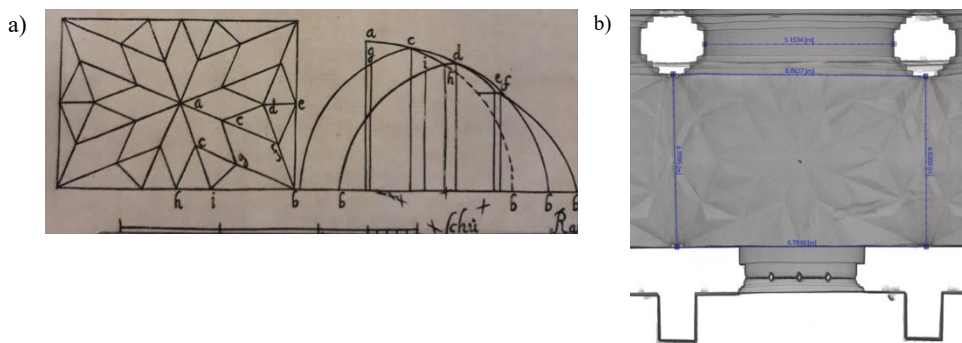


Fig. 4. Diamond vault from the Church of St. Peter and St. Paul in Gdańsk: a) vault plan with construction [source: [13]]; b) scan of a selected bay with its dimensions provided [source: documentation prepared by Aurea Porta Monument Conservation Tomasz Korzeniowski]

Scanning analysis of all bays in the northeastern nave of the church indicates their varying dimensions. The outer bays are slightly narrower than the three central bays of identical length. The inner bays have dimensions similar to those in B. Ranisch's work. Therefore, the plan of the central bay with the diamond vault from the northeastern nave served as the basis for the construction of the virtual model later. Using vault scanning, the average dimensions of an individual bay were assumed: 682 x 467 cm (Fig. 4b).

The centerpiece of the bay's plan is an eight-pointed star, whose arms define the width of the nave. It consists of eight identical rhombuses. The vault's plan was started with the drawing of this star. The quadrature method, common throughout Gothic architecture, was used for this purpose. The quadrature method is described by Matthäus Roritzer in his work from around 1497 entitled *Geometria Deutsch*, thus dating from the Gothic period [32]. This method has been used by many researchers, including François Bucher in his 1972 work *Medieval Architectural Design Methods, 800-1560* [33] and Werner Müller in his *Grundlagen gotischer Bautechnik*, published in 1990 [34]. The starting point for this method was two identical squares, rotated relative to each other by 45° (black in Fig. 5a). The side of the square defines the width of the nave (467 cm). The next pair of squares is created by connecting the points that are the midpoints of the sides

of the black squares (green in Fig. 5a). The final pair of squares is obtained by connecting the midpoints of the sides of the green squares (blue in Fig. 5a). The arms of the star connect the vertices of the green squares (or the midpoints of the sides of the starting black squares) with the eight intersection points of the blue squares (Fig. 5b). The star design also includes eight internal segments connecting its arms to the centers of the squares.

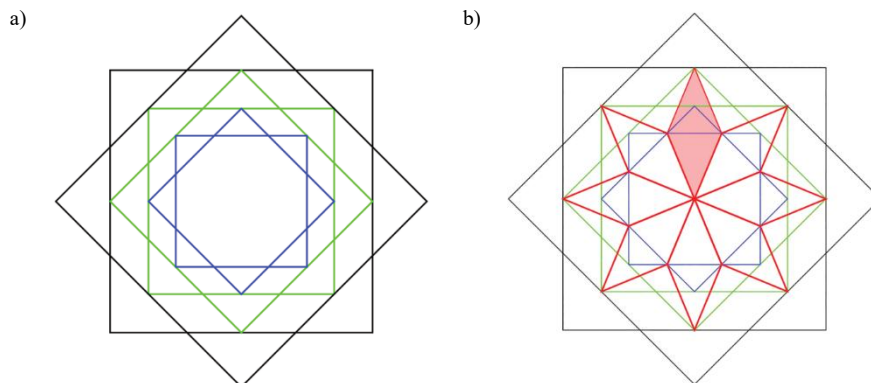


Fig. 5. Construction of an eight-pointed star based on the quadrature method: a) six squares connected using the quadrature method; b) an eight-pointed star obtained from the quadrature

The next step in drawing the vault's plan was to determine its length. Analysis of Ranisch's plan (Fig. 4) shows that this length can be obtained by extending the appropriate arms of the star to their intersection with horizontal lines (Fig. 6a). Connecting the corner points with each other and with the nearest vertices of the star gives all the segments emanating from the corners (Fig. 6b). Extending all the inner segments of the star to their intersection with the nearest lines leads to the next elements of the vault plan (Fig. 6c). The final step was to determine the outermost segments as shown in Figure 6d.

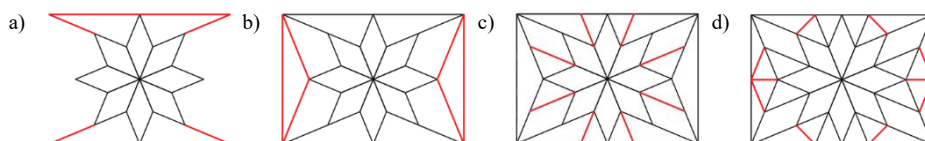


Fig. 6. Constructing the plan of the diamond vault: a) determining the bay length; b) defining the corner line; c) extending the inner sections; and d) drawing the final outer sections

The resulting plan of the diamond vault is a geometrically precise construction based on two basic assumptions. The first one is that the eight-pointed star consists of eight identical rhombuses. The second assumption is related to the fact that the inner lines running parallel to the corresponding inner segments of the vault meet at the corner. These assumptions result in a vault with precisely defined dimensions. Assuming the nave width is 467 cm, the resulting length is 660.44 cm.

After analyzing the diamond vaults in the Church of St. Peter and St. Paul in Gdańsk, the vault plan shown in Figure 6 was verified (Fig. 7). The section marked by Ranisch *de* was eliminated (in Fig. 7, the capital letters A, B, ... indicate the points connecting the individual vault lines – the keystones). The capital letters correspond exactly to the lowercase letters in Barthel Ranisch's notation (Fig. 4). Segment GF was added, and segment GI, which appears only in the southwest aisle, was eliminated. Furthermore, there is no wall segment BH.

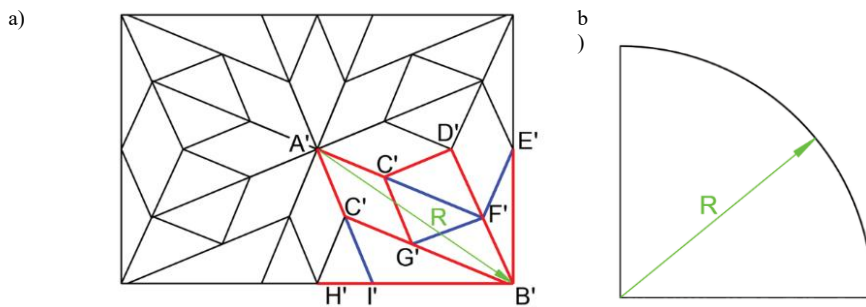


Fig. 7. Plan of the diamond vault: a) with identified nodal points and radius R; b) quadrant with radius R

Wireframe Model of the Vault

To determine the 3D model of a crystal vault, it is necessary to know the geometric principles governing its construction. The construction described by B. Ranisch (Fig. 4a) was used for this purpose. B. Ranisch included a description of the construction of a crystal vault (Num. III) in chapter LXIV of his work (pp. 66, 67). It was assumed that all ridges between the pyramidal cells have the same curvature. They are arcs of a circle with the same radius R. Its size is defined by half of the diagonal of the bay. When determining the radius R, special care must be taken. As Ranisch himself observes (when referring to the crystal vault Num XV in St. Mary's Church in Gdańsk), "*weil keine ganze Büge in den Winkel aus dem Centro gehet*" (translated as "because no rib connects the corner with the center of the vault"). The same observation applies to the discussed diamond vault. No ridge runs along the diagonal. In Figure 7a, the arcs whose construction was presented by Ranisch are marked in red. This description includes determining the heights of all nodal points. Blue lines (not described by Ranisch) were drawn by connecting the points (of known height) with an arc of radius R.

The basis for determining the curvature of ridges is a quarter of a circle with radius R, called a quadrant according to Ranisch's nomenclature (Fig. 7b). Determining the curvature of ribs using a circular arc of a single radius is also called the Prinzipalbogen, Haupt-Bogen, or Quadrant principle [35]-[39]. In addition to the circular arc, two lines emanating from its center, horizontal and vertical, play an important role. The horizontal line is also called the *impost level* in specialist literature [40]-[43]. On the horizontal line, segments will be measured defining the position of vertical lines related to the heights of individual nodal points. The order in which the individual arcs are determined follows the order described by B. Ranisch. Therefore, arc AC and the height of point C were determined first. For this purpose, segment A'C' was measured on the vault plan (Fig. 7a), and an arc centered at point A was drawn with radius $R_{A'C'} = A'C'$ to the intersection with the quadrant (Fig. 8a). The intersection point C determined the curvature of arc AC. The height of point C is the distance C'C. When determining point C, the distance A'C' can be measured alternatively on a horizontal line, as described by B. Ranisch [13], and a vertical line can be drawn to the intersection with the quadrant. Geometrically, two different heights of point C are obtained. However, this difference is negligible. For a nave width of 467 cm, the difference is merely 0.51 cm.

To determine the height of the next nodal point D and the arch CD, it is necessary to measure two distances on the vault plan (Fig. 7a). These are segments C'D' and A'D'. Distance C'D' was measured on the horizontal line of the quadrant (red in Fig. 8b), and a vertical line was drawn through point D'. Using radius $R_{A'D'} = A'D'$, an arc was drawn with center at point A to intersect the quadrant. A horizontal line was drawn from the intersection point P, which, at its intersection with the previously drawn vertical line, determines the height of point D. To determine the arc CD, two arcs were drawn with radius R and centers at points C and D. Their intersection, designated as S (Fig. 8c), is the center of the constructed arc CD (red).

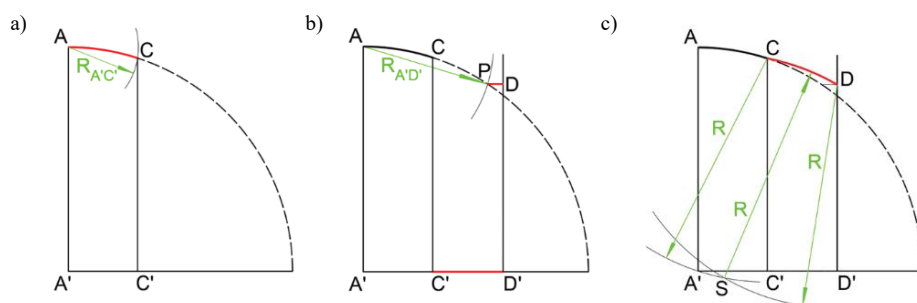


Fig. 8. Defining arcs AC and CD: a) arc AC and the height of point C; b) the height of point D (equal to D'D) determined using two quantities A'C' and A'D', and c) a circular arc of radius R connecting points CD (red).

In a similar manner to the method used for the arch CD, the height of point E was determined, and the arc DE was determined (Fig. 9a). By measuring the segment E'B' on the horizontal line of the quadrant, the position of point B (B'=B) was determined. By connecting points B and E with a circular arc of radius R, the curvature EB was obtained (Fig. 9a). Arches CB, DB, and HB are characterized by the presence of additional nodal points. These are points G, I, and F, respectively. To serve as an example, arc CB was determined with its associated point, G. Distance C'B' was measured on a horizontal line, obtaining the location of point B (Fig. 9b). Connecting points C and B with an arc of a circle of radius R gives arc CB. Point G has the same height as point D. Therefore, a horizontal line was drawn between points G and D. This line, at the point of intersection with arc BC, determined the height of point G equal to G'G.

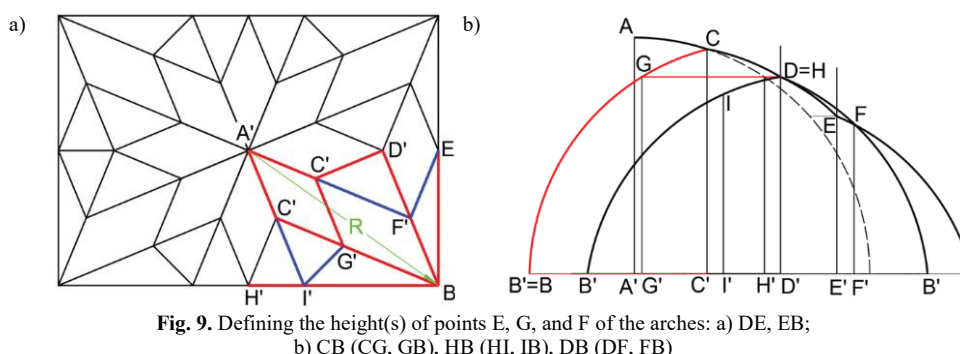


Fig. 9. Defining the height(s) of points E, G, and F of the arches: a) DE, EB; b) CB (CG, GB), HB (HI, IB), DB (DF, FB)

The vault arches marked in blue on the plan (Fig. 7) were determined by drawing circular arcs of radius R through the nodal points, whose heights were determined earlier (Fig. 10).

The data regarding the heights of the nodal points and the curvature of the individual arches were used to construct the wireframe model (Fig. 11a).

The plans of the diamond vaults in the Church of St. Peter and St. Paul in Gdańsk, available in several studies, differ when it comes to the smaller arches. Maria Brykowska referred to the vault plan presented by Barthel Ranisch in her study [30]. She marked the vault ridges with solid lines and the lines between the pyramidal surfaces with dashed lines. The vault plan proposed by M. Brykowska can be found in the work of Milady and Oldrich Rada [1]. A sketch of her plan was also included in an article on diamond vaults by Ł. Drobiec [44]. Post-war documentation illustrating the state of destruction and the progressive degradation of the church shows the actual vaults [31]. They illustrate the vaults in a way that most closely matches what can be seen in the church interior (Fig. 3). When comparing with Ranisch's plan, the existing diamond vault in the northeastern nave lacks ridges DE, GI, and BH (BI, HI). The arch DE is not present in the vault seen inside the church.

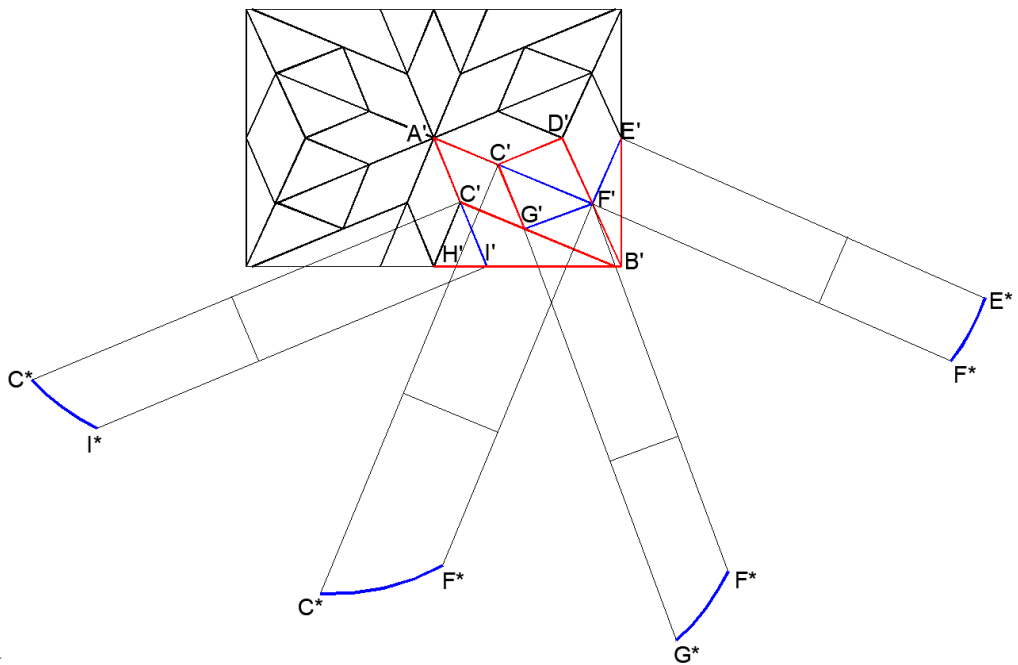


Fig. 10. Arch sections CI, GI, EF, CF, and CH, defined by connecting the endpoints with arcs of radius R

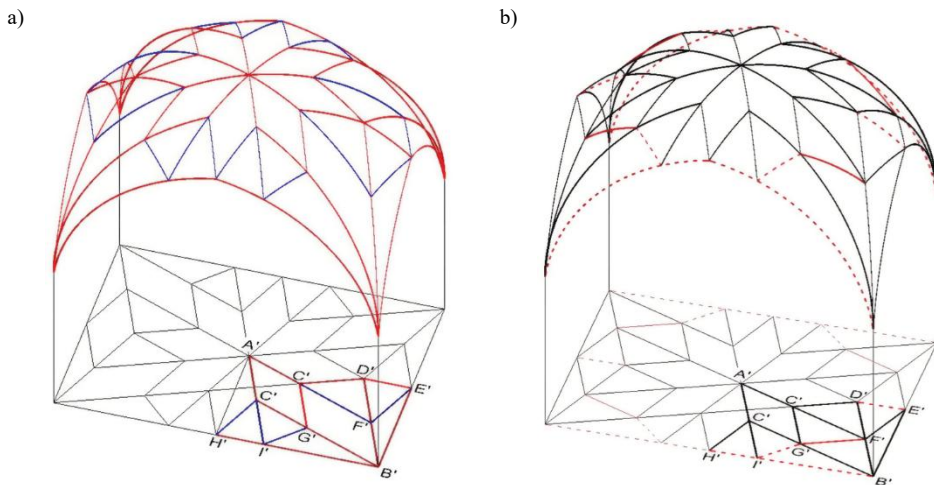


Fig. 11. Wire model of the diamond vault from the Church of St. Peter and St. Paul in Gdańsk: a) construction created in accordance with B. Ranisch's plan; b) model that takes into account the current state of the vault

The plan of the Church of St. Peter and Paul, published by the National Heritage Institute on the *Zabytek.pl* website, does not include it, either. It is also absent from post-war documentation from the years 1945-47 [31]. These studies show the arch GI in the diamond vaults of the southwest nave. Post-war documentation and the preserved remains of the vaults (which collapsed in the years following the war) allowed for their reconstruction. The ridged GI, which now can be seen among the other vault arches, was considered. The new ridge FG should also be considered. The diamond vaults of both the northeast and southwest naves lack wall arches BH (BI, HI). Ranisch's construction of the wall arch was used to determine the heights of points H

and I. In Figure 11b, red dashed lines indicate the elements that were eliminated. The solid dashed line indicates the new arch GF (Points F and G, with previously determined heights, were connected by an arc of radius R). Therefore, the final wireframe model of the vault (which served as the basis for further modeling) was based on B. Ranisch's construction, taking into account the existing diamond vault in the northeastern nave.

3D Model of the Vault

The 3D model of the vault was constructed using Rhinoceros and Grasshopper programs. In particular, Grasshopper allowed for defining an algorithm that generated the object's geometry in individual steps. This approach allows for creating numerous variants of the vault model for different nodal point locations in space. This method also helped with the analysis and better understanding of the principles of forming pyramidal cells.

The algorithm's operation is presented for one central section of the vault with a quadrangular plan. The first step involved selecting the vault points representing the vertices of the pyramidal cells. In Figure 12a, these points are located on the edges of a prism created by vertically extruding the vault projection. Then, arcs connecting these points were drawn on vertical planes defined by pairs of successive points. Their radius R is a constant, equal to half the diagonal of the vault projection (Fig. 12b). In the next step, conical surfaces based on the arcs were generated (Fig. 12c). Their inclination was set by default to be 45 degrees relative to the vertical plane. The algorithm had the ability to change later the angle of the inclination of the cells.

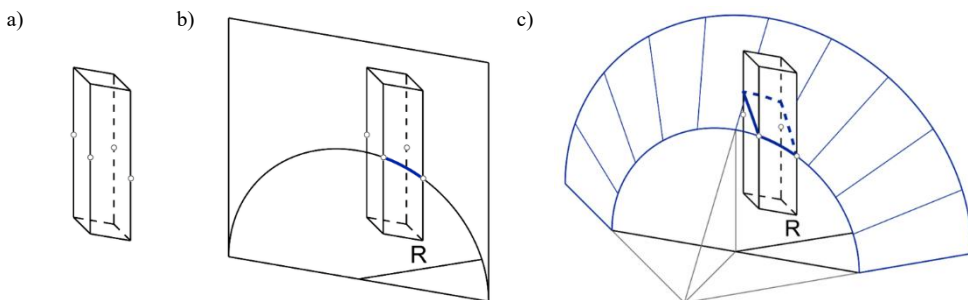


Fig. 12. Modeling a pyramidal cell: a) indicating four vertices; b) arc with radius R connecting two points; c) conical surface based on the arc

The further part of the algorithm is based on Boolean operations on solids. Modeled conical surfaces were extruded vertically downward to create solids. These were used to cut off the lower parts of the prisms (Fig. 13a). The obtained results of the subtraction of solids were summed, resulting in a model of the interior of the pyramidal cell (Fig. 13b). Eightfold duplication of the obtained segment generated a model of the central star of the diamond vault (Fig. 13c). The presented procedure was applied to subsequent sections of the vault. By combining the obtained results and utilizing the symmetry of the studied object, a complete model was created. In addition to the ability to control the inclination of the selected cells, the algorithm also offers the option of skipping the generation of the cell surface for the selected arch. This option allowed the creation in the model of those sections of the vault that rest against the wall.

Six-point cloud cross-sections of the vault were created and compared with cross-sections of the model based on the wireframe model (Fig. 14a). Analysis of the drawings indicated a convergence between the theoretical model and the actual model in the central part of the vault but a significant discrepancy in the inter-bay area. Therefore, another modification was made, increasing the height of the points: D by 25 cm and E by 90 cm (Fig. 11). This change resulted in a good match between the scanned cross-sections of the vault and the computer model (Fig. 14b).

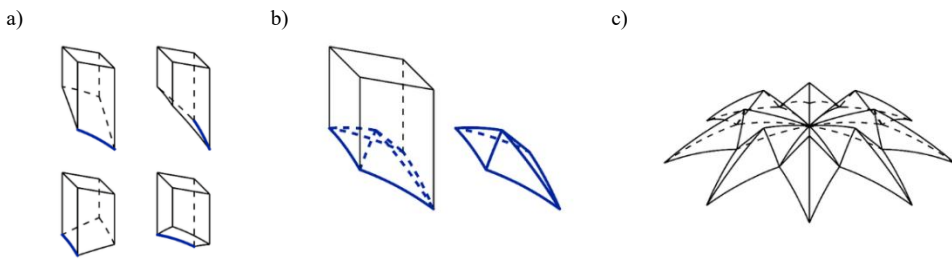


Fig. 13. Modeling of central cell elements: a) cutting with conical surfaces; b) sum of solids; c) central star of the diamond vault

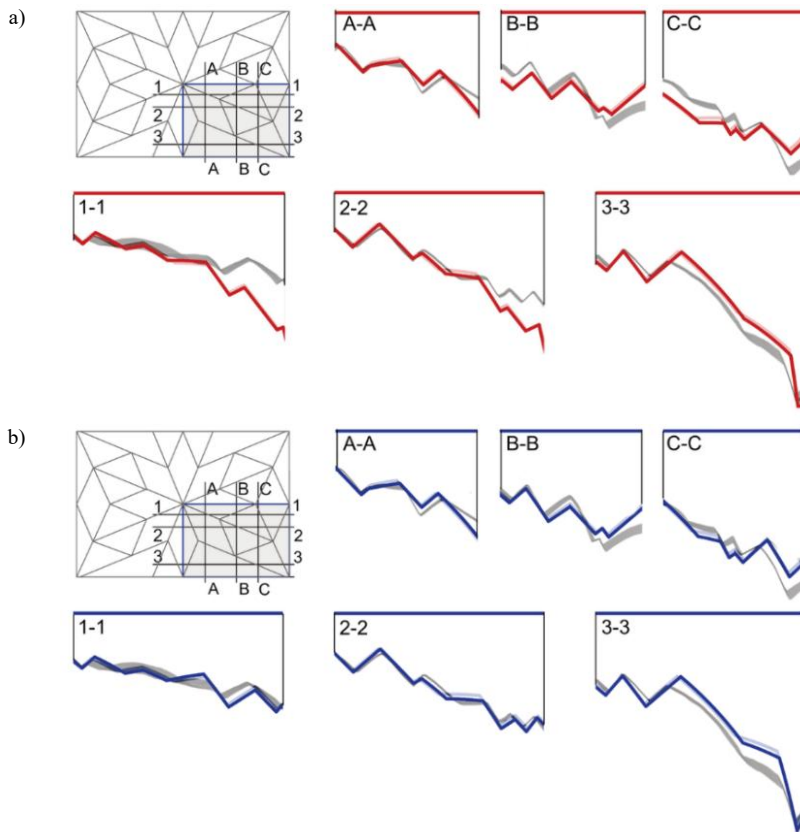


Fig. 14. Comparison of cross-sections of the vault's point cloud: a) with a model of the vault constructed using the wireframe construction; b) with a model created after modifying the heights of points D and E

In particular, to achieve the most faithful representation of the geometric model with the available documentation in the form of a point cloud, individual modifications to the inclination of the particular vault cells were introduced (Fig. 15). The occurrence of varying angles of inclination of the cells was noted by Arkadiusz Staniszewski, who documented the work progress during the renovation and reconstruction of the church [45]. It was observed that not every set of points could generate pyramidal shapes. As the difference in point heights increased for the same radius R , greater inclinations of vault cell surfaces were required to achieve the correct geometry.

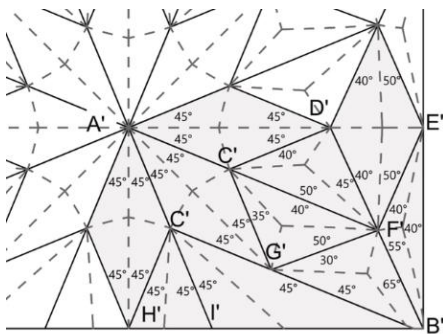


Fig. 15. Angles of inclination of cell surfaces



Fig. 16. Perspective view of the modeled vault

The applied parametric-algorithmic approach allows for the modeling of pyramidal vault cells in accordance with the presented principle (Fig. 16). The script is universal and can be used for diamond vaults with different plans (Fig. 17). The parameterized values in the algorithm include nodal points (a minimum of 3 points defining a convex polygon on the plan), the radius of the arches, and the inclination of individual surfaces. It is also possible to convert the inclined cell surface into a vertical wall.

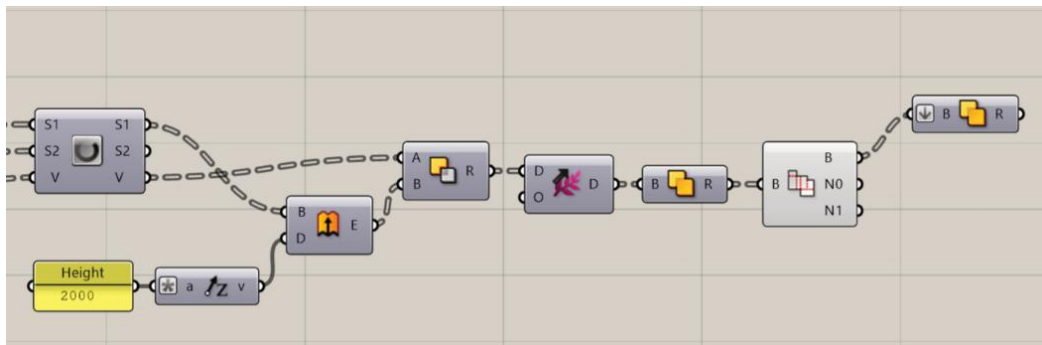


Fig. 17. Fragment of a script with Boolean operations related to vault modeling

The final result of modeling is a digital vault with geometry similar to the actual object (Figs. 16, 18a). The recreated structure displays the same spatial qualities as the original (Fig. 18b).

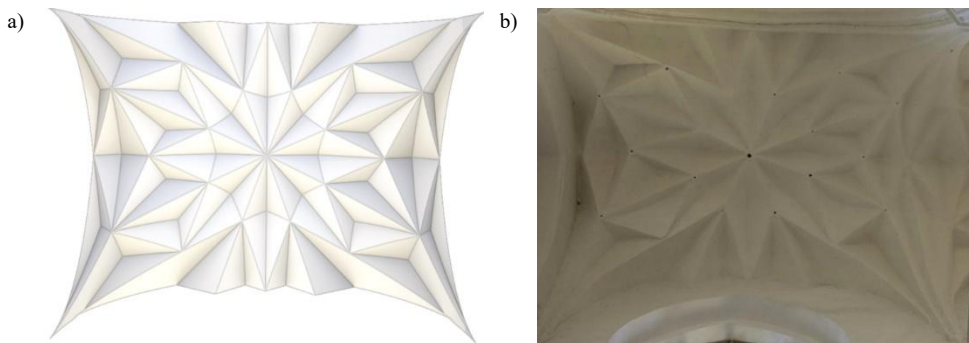


Fig. 18. Comparison of vaults: a) theoretical model; b) photographic documentation

Conclusions

The history of the Church of St. Peter and St. Paul, like many historic buildings in Gdańsk, is very complex and often dramatic. Wartime destruction and post-war deterioration resulted in the collapse of many vaults. The full reconstruction was not completed until 2006, i.e., approximately half a century later. A fire that broke out in the church in 2019 consumed nearly 200 square meters of the roof, but, fortunately, it did not threaten the interior or the vaults. These events were an impulse to conduct a comprehensive laser scan of the entire structure. Point cloud documentation is an invaluable source of information and currently a popular method of documenting data regarding historic buildings [46]-[50]. It is a record of the condition the building is in, primarily enabling the monitoring of any changes. The scan can also be used to conduct various research and analyses of the structure. Due to the numerous deformations to which the interior of the church was subjected (the vaults in the remaining naves largely collapsed), it is not a source of information about the geometric rules governing the construction. Without knowledge of these principles, it is difficult to carry out repair work, especially reconstruction. Therefore, this study focuses on constructing a theoretical model based on principles that the builders could have applied when constructing the crystal vaults. For the first time, the construction record provided by B. Ranisch was verified. Therefore, the resulting vault model is not a digital twin reflecting the actual state of the structure. Many churches in Gdańsk with Gothic crystal vaults have a digital inventory in the form of a point cloud. Their spatial structure was also described by B. Ranisch. This opens the possibility of further research to verify the geometric rules used in their construction. This involves the development of virtual models, combined with vault scanning.

There are certain differences in the construction of the diamond vaults in the side naves of the Church of St. Peter and St. Paul in Gdańsk. Each of the 14 bays has different dimensions. Hence, there are discrepancies between the model based on B. Ranisch's notes and the model referring to the bay selected from the original northeastern nave (Fig. 11). The central parts of the theoretical structure matched the vault scan very well. This convergence was also confirmed in the wall areas. The greatest discrepancies in modeling occurred in the inter-bay areas. Maintaining a constant angle of inclination of the vault cells (45 degrees) for the wire structure built in accordance with B. Ranisch's notes was geometrically impossible. Two methods of implementing cells in this problematic area were tested: changing the angles of inclination (even up to 75 degrees) and raising the height of several nodal points. Comparing the results with the vault scan indicated that raising the height of two points in the inter-bay area produced good results. The final model is ultimately based on the diagrams from old prints and refers to the actual state of the vault.

Diamond vaults are structures with complex geometry and are difficult in modeling. The use of algorithmic and parametric tools not only allowed for the reconstruction of their shape but also unified operations. The developed algorithms, due to the variation of many parameters, can be applied to design other types of diamond vaults. Consequently, this opens up opportunities for further research of a theoretical nature as well as conservation work.

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