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METHODOLOGY IMPOSITIONE PRIMARII LAPIDIS: THE CATHEDRAL OF SANTA MARIA DE TORTOSA

Cinta LLUIS-TERUEL[1,*], Josep LLUIS-GINOVART[1]

[1] Universitat Internacional de Catalunya – C/Inmaculada 22, 08022 Barcelona (España)

Abstract

The Gothic cathedral of Santa María de Tortosa (1347) is built on the Romanesque (1158), and this, in turn, on the grounds of the Turtusha mosque (c.717), and this, on the Roman building of Dertosa (c.100), oriented Ac.100 = 108, 64º, being compatible with the direction to the Kaaba in Mecca Ac.717 = 108, 25º. The Romanesque cathedral causes a redirection on the existing remains, with a more probable orientation A1158-Ii = 95, 50º and range (93.00º-97, 99º), whose interval is between the canonical orientation from East to West, and in the topographic criterion of direction towards the top of the Creu (383m) ACreu = 97, 76º. The Gothic cathedral A1347 = 98, 17º is geometrically reconsidered on the Romanesque one as a ring road, justifying its difference.

***Keywords**: Church orientation; Christian Religion; Cathedral; Gothic; Romanesque; Tortosa*

Introduction

Nicolas IV (1288-1292) reformed the primitive facade of Santa María la Maggiore, in which Filippo Rusuti (c. 1255 - 1325 circa), made mosaics (1288-1292) representing Pope Liberius (352-366) tracing on the snow on August 5, 358, the plan of the basilica with the inscription Virgo Mario appa-ruit PP Liberio dicens: fac mihi ecclesiam in monte Supera-gio sicut nix indicat [1]. The importance of the rite was established in the Ceremoniale Episcoporum, in the prints of the Pontificale Romanum (1595) by Clement VIII (1592-1605), De benedictione and impositione primarii lapidis pro Ecclesi aedificada [2] (Fig. 1).

Saint Isidore of Seville (c. 556-636), in the Originum sive etymologiarum libri viginti (c. 630) (Etym. L XIX. VIII) determined the simile between the architect when he traces the foundation and the meaning of Peter, as a symbol on which the church site [3]. The orientation of the sacred buildings of Europe in the centuries (IX-XIV) has been the subject of study: in Austria, C. Firneis and M.G. Köberl (1989) [4]; Denmark, N. Abrahamsen (1992) [5]; Slovenia, S. Čaval (2009) [6]; England, P.G. Hoare and C.S. Sweet (2000) [7] and S.C. McCluskey (2007) [8] and in Italy, G. Roman (1992) [9] and M. Incerti (2013) [10]. In the case of Spain, it has been the Mozarabic churches (2018) [11], the pre-Romanesque of Asturias (2015) [12], those of the Camino de Santiago (1998) [13], the Romanesque of Galicia (2105) [14], the one from the Aran Valley (2019) [15] and that of the Boí Valley (2019) [16].

The orientation of the sacred buildings

The orientation of the apse arranged on the east was imposed in the constructions of the centuries (IX-XIII) carried out under the influence of the liturgies of the time, based on Liber officialis (820-826) by Amalario de Metz (c.780- 851), which inspired the main Romanesque

* Corresponding author: cintalluis@uic.es

canons, from Rupert de Deutz (c. 1075-1129) with his Liber de divinis officiis (1120). The considerations on the orientation are established in the Gemma animae (c. 1120) of Honorius of Autun (1080-1153) considering it in the ecclesiae ad orientem vertuntur ubi sol oritur (Gem. Ani. I, 129, De situ ecclesiae). In the Mitralis of Officio (1190) of Sicardo, bishop of Cremona (1185-1215), is considered in the Ad orientem, id est, ortum solis aequinoctialem Miter. I, 2, De fundatione ecclesiae). In the treatise of Jean Beleth (fl. 1135-1182) with his Rationale divinorum officiorum, (c. 1150) is specified in Versus sim orientem, hoc est, versus solis ortum aequinoctialem (Ration. Ch. II, De loco). Finally, this theme is collected in Prochiron, vulgo rationale divinorum officiorum (1291) by Guillermo de Durando (1230-1296), Yerus ortum solis aequinoctialem, ad denotandum quod ecclesia quae in terris militât (Prochi. I, 8, p. 5) considered a work of great influence in the Gothic world.

Fig. 1. De benedictione & impositione primarii lapidis pro Ecclesia aedificada.
Pontificale Romanum, Pio IIII Pont. Max.Venetiis, Apud Iuntas, MDLXI. Tortosa Chapter File (ACTo)

There are direct references to the existence of these liturgies in the Chapter Library of the Cathedral of Tortosa: this is the case of Guillermo de Durando's Prochiron in the codex (ACTo 58), (f. XIII) one of the first existing copies. The incunabula of the edition of Rome (1477) (ACTo 258) and Venice (1482) (ACTo 290), by the same author, four codlibetos (ACTo 42) and by Iohannis Belleth: de ecclesiasticis officiis (ACTo 219) (XIII), by Honorio de Autún the (Act 122). The bishops and canonicals knew the outline of the Ecclesiae ad orientem vertuntur ubi sol oritur.

The aim of the research is to find a methodology to determine in cases where no excavations have been carried out the hypotheses of the orientation process of buildings with overlapping structures. The objective is to determine how the Cathedral of Santa María de Tortosa was oriented, and with what criteria. For this reason, various hypotheses have been established:

- Canonical criterion. It consists of assuming the orientation according to the liturgical treatises, placing the apses from East to West. The canonical orientation is heir to the Roman tradition of solar course, east-west, M. Delcor (1987) [17].

- Liturgical calendar criterion. It consists in supposing that the church was oriented on the day of the feast of the saint to whom it was to be dedicated, W. Johnson (1912) [18], H. Benson (1956) [19] and E. Spinazzè, 2016 [20].
- Paschal calendar criterion. It consists in supposing that the churches were oriented on the Easter festivity of the year of their construction, the Easter of Resurrection since the Council of Nicea (325) was fixed as the Sunday after the spring full moon, G. Romano (1997) [21] and S.C. McCluscKey (1998) [22].
- Topographic criteria. It consists in assuming that the orientation of the church is influenced by the environment, either because it is linked to a geographical feature or a characteristic landscape, A.B. Knapp and W. Ashmore (1999) [23] and A. Sassin (2016) [24].
- Constructive criteria. It consists in assuming that the church was oriented on the day of its redefinition according to adequate construction conditions, not depending on any liturgical aspect, J. Pérez Valcárcel (2018) [25].

Cathedral of Santa Maria de Tortosa

In the 14th century, the city of Tortosa was located in the center of a large and prosperous diocese, located at the mouth of the Ebro River and equidistant from Barcelona and Valencia 200km. In 1347 the first stone of the new Gothic cathedral was laid, which replaced an old Romanesque construction consecrated in 1178 and begun in 1158. The location and orientation of the Romanesque cathedral suggested, from a documentary point of view, two plausible hypotheses. José Matamoros Sancho (1866-1937), in La Catedral de Tortosa (1932), collecting the historiographic tradition, placed it at the foot of the current basilica, perpendicular to the current axis, facing North-South. On the contrary, Monsignor Aurelio Querol Lor (1917-1994), in the Official Program of Holy Week (1992) and V. Almuni in the Romanesque Cathedral of Tortosa. Documentary approach to its history (2004), they do it in a similar arrangement to the current cathedral, oriented from East-West. In turn, this Romanesque construction was located on the land occupied by the Turtusha mosque (c. 717), verified by A. Virgili in the Diplomatari de la catedral de Tortosa (1062-1193)/(1997).

The results of the geophysical survey of the cathedral's subsoil carried out in the campaign (2012-2013), together with the analysis of the topographic techniques (2013-2015) of the imprints of the elements prior to the construction of the Gothic factory, made it possible to provide complementary data to the documentary studies and determined an E-W orientation. These data allowed to fix some important elements of the Romanesque cathedral and also auxiliary elements of the Gothic, showing an orientation of the East West cathedral, inscribed and centered on the current Gothic cathedral, but without determining its Azimuth. The objective of this research is to determine what are the possible orientations, azimuths (A) of the different overlapped structures, as well as, if possible, the method of tracing the established hypotheses.

The construction of the cathedral of Santa María Tortosa (1347-1441)

In 1345 the Chapter entrusted the construction of the cathedral to the magister opera Bernat Dalguaire (1347). The first stone was laid on May 21, 1347; after that and with the arrival of the Black Death (1347-1348) and the War of the two Pedros (1356-1369) the work suffered a standstill. The head of the new cathedral of Tortosa, with a heptagonal plan, was raised and covered between 1374 and 1441.

In a first phase, the construction of the belt of radial chapels, covered in a correlative and sequential way, from the Gospel sector to the Epistle sector, between 1383 and 1424 was arranged. The initial section of proportion 9/5, relatively low and unusual at the end of the 14th century, it is justified by the elimination of the existing walls between the radial chapels. The second phase will determine the construction of the ambulatory (1424-1435). At this time there is an increase in section due to the change to a sesquitercia metric (9/6).

Unlike the construction sequence of the chapels, the vaults of the ambulatory were covered symmetrically (1432-1434) on the axis of the presbytery, closing from the mouth of the latter towards the interior. The third and last phase corresponds to the covering of the presbytery (1435-1441) with the placement of the great keystone and the immediate closing of the vaults. This process is carried out with both constructions coexisting in the same space, in a continuous process of disassembling the Romanesque construction and the construction of the Gothic one (Fig. 2).

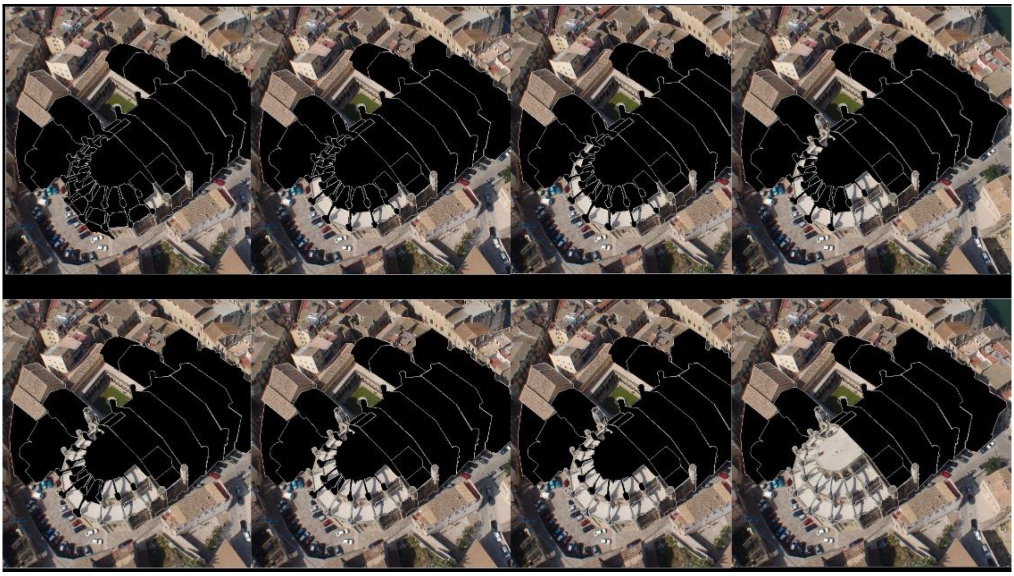

Fig. 2. Cathedral apse construction (1347-1441)

The Cathedral of Santa Maria (1158-1437)

The consecration and endowment of the Romanesque cathedral dates to November 28, 1178 and its beginning dates back to the beginning of the year 1158. From a historiographical point of view, we must go back to the First Col·loqui Cristofor Despuig (c. 1510 - 1574) of the Col loquis of the famous city of Tortosa (1575), where it determines the coexistence of the Romanesque cathedral with the new Gothic cathedral. The same is done by Francesc Martorell de Luna (1586 -1640), in the History of ancient Hibera (1626) and the codex ancient notes of this cathedral (c.1645) by Canon Manuel Macip (f.1644). The remains of the facade of the Romanesque cathedral are not dismantled until 1703. The facade is perfectly visible in Antoon van den Wyngaerde's (1525-1571) View of the City of Tortosa (1563) from the Osterreichische Nationalbibliothek, Vienna, Picture Archive, Cod. min 41, fol. 7r, (Fig. 3).

The passage of time erases the presence of the remains of the Romanesque cathedral and causes the ecclesia vetulam to be forgotten. Thus, in Volume V of the literary Journey to the Churches of Spain (1803-1852) by Joaquín Lorenzo Villanueva y Astengo (1757-1837) and Jaime Villanueva Astengo (1765-1824), they placed the Romanesque cathedral at the foot of the current basilica. The evidence of this construction was found in November 2002 during the archaeological control of the works Structural contention of the mur of the main façana of the Cathedral of Tortosa (2002), finding two vault keys embedded in the base of the main facade of the 17th century cathedral (Fig. 4).

Fig. 3. Detail of the west cathedral facade of Tortosa (1563). Antoon van den Wyngaerde (1525-1571). Osterreichische Nationalbibliothek, Viena. Picture Archive, Cod.min 41 (fol. 8 r).

Fig. 4. Romanesque cathedral vault keys (1158-1437)

The Turtusha Mosque (c.717-1158)

Within the illustrated literary tradition, Manuel Risco (1755-1801) in Sacred Spain. Geographical-historical theater of the Church of Spain (1747-1801) published different news in reference to the mosque and the consecration ceremony of the Romanesque cathedral. The Romanesque construction occupied the old mosque referred to as “quam habeo in villa Dertose ante ecclesiam vetulam. Que affrontat a parte orientis in carraria publica ante predictam ecclesiam et meridie in domibus”. At the foot of the current nave, Cristofor Despuig describes the meeting of the remains in 1552, which he identifies as a Roman Temple, but not archaeological evidence of the mosque (c.771-1158) is available to date.

Methodology

To determine the alignments, are taken as a basis the following topographic surveys:

Campaign (1995-2000) (L1) A first computer topography of the cathedral was carried out between (1995-2000). The survey was manual and compensated by polygonal points fixed by means of a SOKKIA, SET 3BII total station of 10cc of angular precision in 2cc resolution mode, equipped with an automatic dual axis compensator and maximum range with a reflector of 2,500m, with precision of +/- (3mm. + 3ppm.). Campaign (2013-2015) (L2). Terrestrial

Laser Scanner (TLS)r campaign using Leica Scan Station C10 (Campaign 2013), instantaneous maximum speed of 50,000 points/sec, dual axis compensator, topographic level viewing range, class 2 laser level (IEC 60825-1). Leica Scan Station P20 (2014-2015 Campaign), 3D accuracy 3mm at 50m; 6mm at 100m, linear error less than or equal to 1mm. Angular precision 8” horizontal/8” vertical. Standard deviation in the acquisition of 2mm targets at 50m, 1” resolution dual axis compensator, +/- 5 'dynamic range, 1.5” precision. wavelength 808 nm (invisible) / 658 nm (visible). Class 1 laser (IEC 60825: 2014) (Fig. 5).

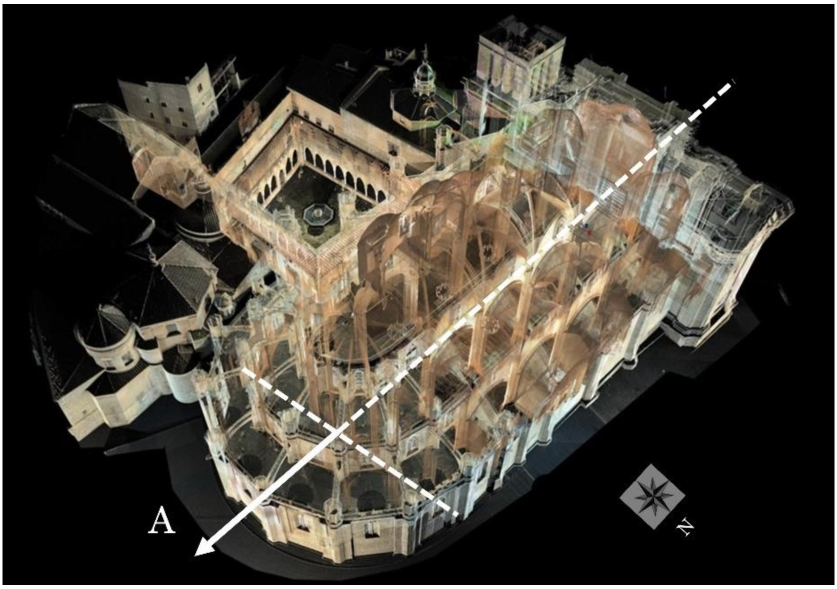

Fig. 5. TLS Results, Campaigns (1995-2015)

The data treatment used to register the point clouds in the same local coordinate system was the Cyclone software, with a standard deviation of 0.003 m. The error considered was 0.009m, resulting from the registration based on the objective, the precision of the device and the registration process. To generate the 3D mesh of the model from the points selected with the 3DReshaper program. The mesh density was adjusted to an average triangle size of 0.025µm. The total error established in the process is within a range (0.010-0.35cm). The data was georeferenced with GPS, according to the UTM coordinates of longitude (λ) and latitude (ϕ), Hayford Ellipsoid FUS-31-N, USER 31, in the cartography of the Cartographic Institute of Catalonia (ICC) (Table 1). The total error established in the process is within a range (0.010-0.035m), which represents a possible deviation of 1/4th.

Table 1. Reference data longitude (λ) and latitude (ϕ)

Element	Coordinates			
	GMS Sexagesimal Coordinates		Decimal coordinates GD	
	(ϕ), La (º ' ")	(λ), Lo (º ' ")	(ϕ), Laº	(λ) , Loº
Cathedral major key	48º 48' 51.413"	0º 31' 21.340"	40.81428149	0.52259448
Center apse wall	40º 48' 51.285"	0º 31' 22.134"	40.81424574	0.52281514
Center facade	40º 48' 51.735"	0º 31' 18.933"	40.81437086	0.52192586
Coll d'Alba	40º 48 ' 57.968"	0º 34' 15.894"	40.81610227	0.57108175
La Creu	40º 48' 35.018"	0º 34' 2.205"	40.80966663	0.56727928
Kaaba La Meca	21º 25' 20.92"	39º 49' 34.197"	21.42249766	39.82616597

To determine and quantify the accuracy of the azimuth orientation (A) the most probable azimuth was determined, taking into account the three possible orientations: the mosque A (c.717), the Romanesque A (1156), the Gothic A (1347), and also the characteristic values of the uprisings (L1) and (L2). To determine the astronomical declination corresponding to the orientation (δ), the height of the horizon (h) was calculated using a calculator [26], conferring a possible error of 1½º.

The appreciation value of the layout for the construction of the cathedral was determined through the basic pattern that is the Cana de los Llibres d'Obra (ACTo), 8 palms (palm) and this is divided into 12 fingers (dits). The measure of the Tortosa cane was defined in Book IX, Rubric 15.5 of the Consuetudines Dertosae (1272). The cathedral cana (1.858m), the palm (0.2323m) and the finger (0.019m) that were taken as a reference of the appreciation value since the finger is the smallest of the length units. The measure of its error is (1.01%) with respect to the Tortosa cana. At present, the evaluation of the uncertainties of the works is 0.025m. If we draw a circumference of a diameter of one cana, the arc of circumference relative to the measurement of the finger will have an angle of 1º, therefore, this will be the error of appreciation.

The value of setting out the factory and taking as reference the Roman settlements and layouts 1º [27], represents in terms of sexagesimal error (1 / 360º = 0.28%).We will consider a precision to assess the different hypotheses of the summation of the methodological processes of the topographic survey ½º, astronomical calculations 1º / 2, the value of appreciation of the lowest measurement of the time 1% and those of the possible stakeout 1º, the verification of the alignments, Σ = (±3.75º) whose interval will be [86.25º-93.75º] for a canonical orientation [E-W = 90º].Some figures have been illustrated with images from Google Earth Pro (Images 14 / VI / 2017) Image Landsat / Copernicus Data SIO, NOAA, US Navy, NGA, GEBCO.GMS, by checking these data, an error of 20.90 m is established, thus the precision of locations and alignments distant to 4000 m, have a precision of (0.3º).

The reference value of the error (± 3.75º) It must take into account the position of the spring equinox in the years (1158 and 1347). The aequus nocte helped to fix the Easter of Resurrection since the Council of Nicea (325). To determine the date of the Patron of the invocation, it must be taken into consideration that the Roman Calendar of Paul VI (1897-1978), from the Mysterii paschalis celebrationem (1969) [28] does not coincide with those of use in the reference period (11th-13th centuries), considering the Hieronymianum Martyrology (6th century) [29]. The change in the date of the Gregorian calendar (1582), of the Inter gravissimas bulla of Pope Gregorio XIII (1502-1585), with the Julian of the study period must also be taken into account [30].

Santa Maria de Tortosa Cathedral orientation

The Gothic cathedral of Santa María de Tortosa (1347) was built on a previous Romanesque one (1158), and this in turn on the land where the Turtusha mosque occupied (c. 717). The superposition of three different frames and metrics can pose possible influences on the orientation criteria of the primitive settlements on the later constructions.

The geometric problem is to trace the apse of the Gothic cathedral, where the seven radial chapels have to be placed on the perimeter of a circumference, the center of which was not accessible, since it was occupied by the presbytery of the Romanesque cathedral. The magister operis had to solve three geometric problems at the time of building a heptagonal apse: the first, the use of some method to trace the heptagon, since it did not appear in the texts of the few practical geometries of the time. Secondly, the geometric figure had to be built without knowing the center, and finally, the proportional relationship between the nine radial chapels, seven located in a curved section of the apse and two in the straight section of the ambulatory and the lateral naves (Fig. 6).

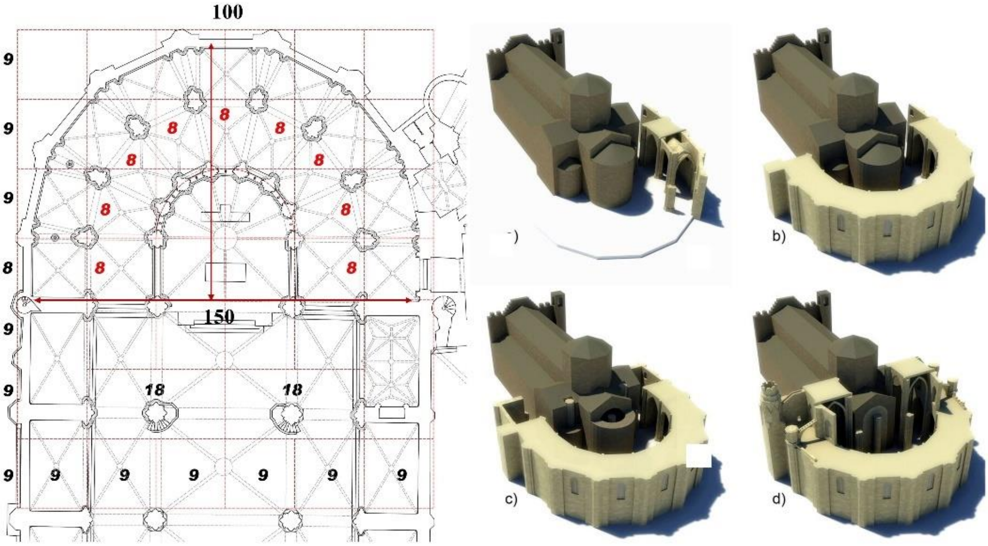

Fig. 6. Method and metrology of the apse of Tortosa (137-1441)

The head of the cathedral has metrological proportions of 150 palms wide, 100 deep and 99 up to the wall; the radial chapels - square in plan - are 21x21 palms. With the help of computer methods, it was determined that the interior points of the pillars, where the work was redesigned, equidistant 24 palms, 3 canas of Tortosa. The metric established in the apse section has 45 palms in the radial chapels, 72 palms in the ambulatory and 100 palms in the presbytery, with a key greater than 10 palms in diameter [31]. In this way, the master knew that by rethinking a radial chapel of 24 palms and once the Romanesque chevet was freed, the opposite chapel would be exactly 54 palms from the center of the presbytery (Fig. 7).

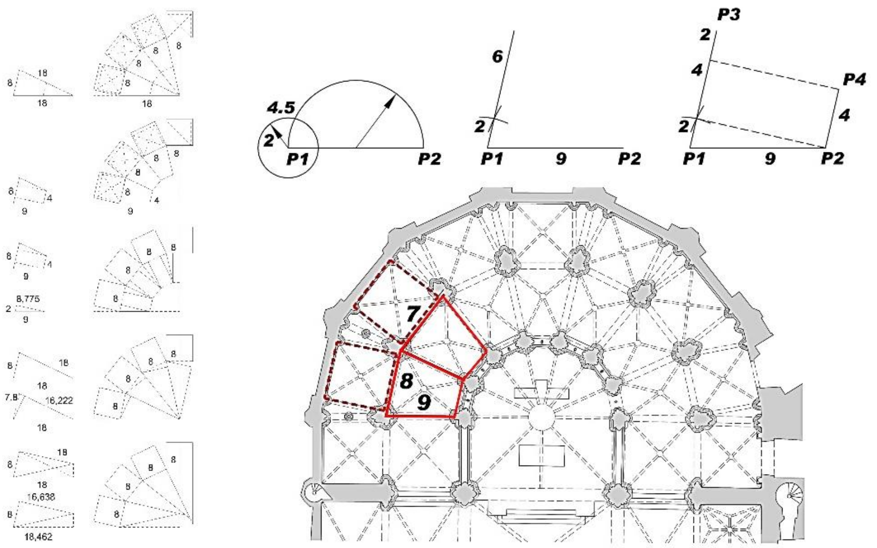

Fig. 7. Methodology to draw the cathedral apse without its center

Thus, a Gothic cathedral could be built over the preceding Romanesque one, without having to provoke the demolition and maintaining divine worship in the cathedral. The difference in the total width of the apse (35.17m) with respect to the metrological width of 150 palms (34.85m) is produced, either by the error of the geometric layout process that circumscribes the Romanesque cathedral, or by fitting to an existing element of this old construction. The orientation was determined by taking the direction from the center of the major key to the middle of the enclosure wall (A1347 = 98.17º) and its range [94.92º-101.92º]. The alignment of the central nave (A = 98.57º) and that of the cathedral including the nave (A = 98.45º).

The Cathedral of Santa María Tortosa (1158)

There are, in the current cathedral, remains of the old Romanesque cathedral or signs of previous constructions, where the environment of the cloister is the demonstration of these transformations, first with the construction of the cathedral nave (1455), then with the Chapel of the Santa Cinta (1672) and the Capilla del Santísimo (1896), where the evidence remains on the constructive limits of this evolution. If we analyze these imprints (Ii):

Imprints 1, (I1). End of the layout of the apse belt. The evidence is of small geometric alterations, produced by the adjustments of the Gothic cathedral with the Romanesque one, which underwent a redirection from the construction of the first Roser chapel (1455-1490). These can be seen at the start of the Santa Catalina staircase (1424) (Fig. 8).

Fig 8. Staircase of Santa Catalina (1424), meeting with the eccelisa vetulam and the Gothic factory.

Imprint 2. (I2). Between the space of the apse and the Chapel of the Blessed Sacrament, the old Chapter House, there is a vaulted space. In the traça de Guarc (c. 1345-1380), there are two grids with a square plan with a ribbed vault. On the Guarc plant we see a rectification in the first radial chapel of the Gospel, initially similar to the typology of the rest and which was later isolated and finished with a ribbed vault. This space was essentially transformed with the Gothic construction of the spiral staircase of the Torre de Santa Caterina (1421). In this area, the declination of the wall with the old alignments is observed (Fig. 9a).

Imprints 3 (I3). The old Chapter House that was dismantled in 1824-1832 and replaced by the construction of the Santísimo chapel, later the seat of the Santo Cristo Parish (1896) (Fig 9b). The Chapter House is dated between 1320 and 1345 of the remains that are preserved are the door and its symmetrical windows. The existing metrology, together with the description by

Villanueva (1806) allows us to determine this set with two central columns and six square ribbed vaults.

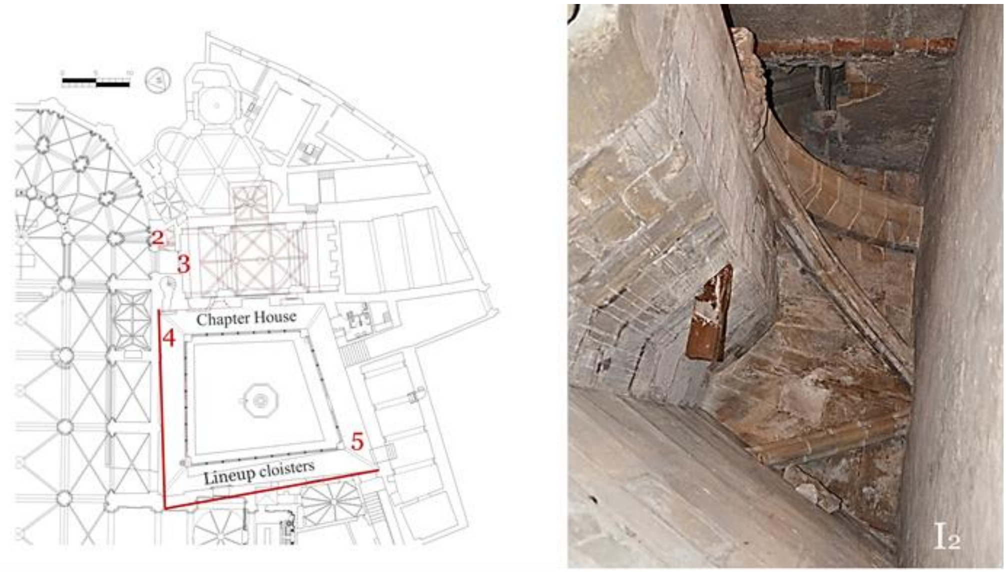

Fig. 9. a) Imprints of the cloister of the Romanesque cathedral; b) Imprint I2 of the old Chapter House

Next to the Chapter House, the chapel of San Juan del Capitulo and later Santa Cándida was built, hired in 1364 by Joan Valença with an orientation (ASc = 5.39º). One hypothesis could be that the Romanesque seo azimuth was perpendicular to the East Gallery (A1158I3 = 95.39º) and to the Chapter House (Fig. 10).

Imprints 4.5 (I4, I5) New Alignment of the Cloister (1455). The metrological pattern of the bulk of the arcade of the cloister changes. The walls of the East (Fig. 11) and South galleries measure 0.35 m, equivalent to a palm and a half of the palm of Tortosa, while those of the North and West are 0.29 m thick, which is equivalent to a palm and a half of the palm of Barcelona and that Felipe II (1527-1598) unified in the Cortes de Monzón (1585), as a metric measure in Catalonia.

The reconstruction of the arcade of the North gallery dates back to 1593, just after the construction of the Chapel of the Name of Jesus. Later, in the West Gallery of the cloister, the area where the Major Priory was located, it underwent a new reorientation with the project that of the Chapel of the Santa Cinta (1672). It is possible to hypothesize that the primitive orientation had the direction of the base imprint (I4) (A1158I4 = 93.11º).

Imprint 6 (I6) Tower of the Chapel of San Pedro. After the start of the work started, the factory suffered a stoppage due to the plague of 1347 and the war of the two Pedros that kept the work process slow until 1375 (Fig. 12).The work was resumed on the part of the northern tower of the Chapel of San Pedro, causing a change in the layout of its geometry perfectly visible at its base, created by the formal arrangement of a circular element, to break with the alignment of the lower geometry of the tower, creating an upper structure in the shape of an octagonal plan. In this context are framed some plans that Andreu Julià made between 1375-1376 of the Seo tower in Lleida. The orientation of the base of the spiral staircase of San Pedro (1383) is (A = 85.00º). The possible hypotheses of the orientation of the imprints; (I3) (A1158-I3 = 95.39º), (I4) (A1158-I4 = 93.11º), and (I6) (A1158-I6 = 98.00º) could preserve the direction of the Romanesque cathedral.

Fig. 10. East Gallery, cloister where the door of the old Chapter House is located

Fig. 11. Romanesque Cathedral North Gallery imprints

Fig. 12. Tower of San Pedro, beginning of the Gothic cathedral (1347)

The Turtusha Mosque Orientation (c.717)

We do not have archaeological evidence of the presence of the mosque, but we can determine the azimuth of the cathedral with respect to the orientation of the Kaaba. The major key coordinates of the Cathedral of Tortosa GMS (N 40 ° 48 '51.413' '; E 0 ° 31' 21.444 ") and those of the Kaaba in Mecca GMS (N 21 ° 25 '20.992' '; E -39 ° 49 '34.197' ') Ac.717 = 108.25 °. Therefore, its verification range is [104.50º-112.00º].

Orientation hypothesis analysis

According to the astronomical calculations of the declination (δ), the dates of the solar visualization on the horizon are established according to the different azimuths (Table 2).

Table 2. Astronomical declination (δ) as a function of azimuth (A)

Cathedral key (ϕ)-(λ), N 48° 48' 51.413"- E 0° 31' 21.340"				
	Azimuth	Height h.	Declination	Time
Element orientation	A (º)	(h) (º)	(δ) (º)	h:m:s
Roman temple A $_{(s.I-II)}$	108.64	4.69	-17.52	7:18:53
Turtusha Mosque A$_{(c.717)}$	?	?	?	?
Romanesque cathedral A$_{(1156)}$	93.11	4.61	-0.81	6:28:13
Gothic cathedral A(1347)	98.17	4.95	-4.02	6:49:58
Coll de l'Alba A$_{(Coll\ Alba)}$	86.92	4.19	4.41	6:51:06
La Creu A$_{(Creu)}$	97.76	5.08	-3.22	6:45:20
Kaaba la Meca A (Kaaba)	108.25	4.69	-17.52	7:18:30
Orientation E-W	90	4.70	2.02	6:23:43

From the different orientation criteria of the sacred buildings, the following is extracted:

- Canonical criterion A = 90º placing the apses from East to West. The orientation of the Romanesque cathedral A1158 with a rankof the three hypotheses (93.11º, 95.38º, 98.00º), with an error between (0.86% -1.50%, 2.22%). Therefore, the azimuth (A1158-I4 = 93.11º) of this hypothesis would be in the range [86.25º-93.75º] for an orientation [E-W = 90º].

- Liturgical calendar criteria. It consists in supposing that the church was oriented on the day of the feast of the Saint to which it was to be dedicated. The Romanesque cathedral is oriented on its horizon on March 20, 1158, whose festivity corresponds, according to the Martyrology Hieronymianum (6th century) at the 14th calendar of March, among others, to San Marcos, Santa Emilia, San Secundino or Santa Victoria. Tradition gives the date of the descent of the Virgen de la Cinta on March 25 in 1178, prior to November 28, 1178 of the consecration of the cathedral, the feast of the Incarnation, the beginning of the calendar of the anno incarnationis.

- Paschal calendar criterion. It consists in supposing that the churches were oriented on the Easter feast of the year 1158 or 1347. The Breviarium Dertusense cum Calendario (ACT 115) (14th century) (Fig. 13), gives for the year 1158, the numerus aureus and the Sunday letters (XIV; D), the date of Easter April 19 (April 26 in the Gregorian calendar), and for 1347 (XVIII; G), the Resurrection April 1 (April 9 in the Gregorian calendar) (Fig. 14).

- The astronomical declination (δ) of the azimuth of the three hypotheses of the Romanesque cathedral, corresponds to the interval 12-20 March, being outside the dates of Easter. The Resurrection must be between March 22 and April 25. For this reason, it is difficult to establish this hypothesis with the criterion of orientation with respect to Easter.

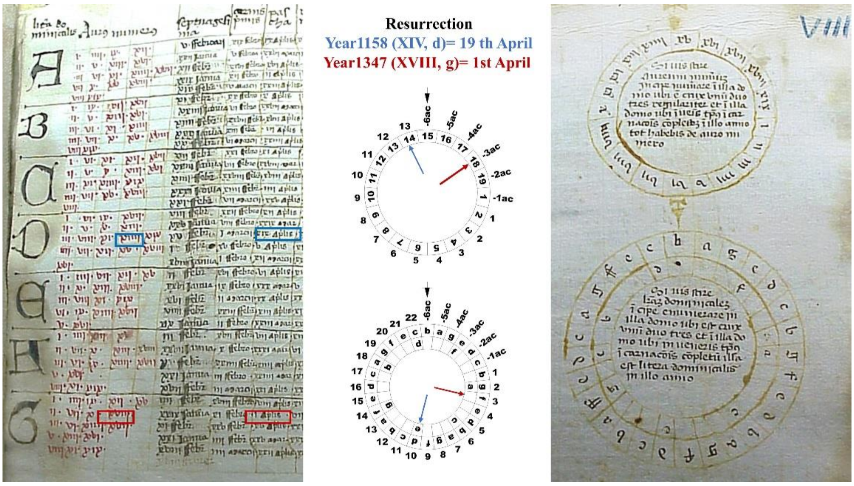

Fig. 13. *Breviarium Dertusense cum Calendario* (ACTo 115)

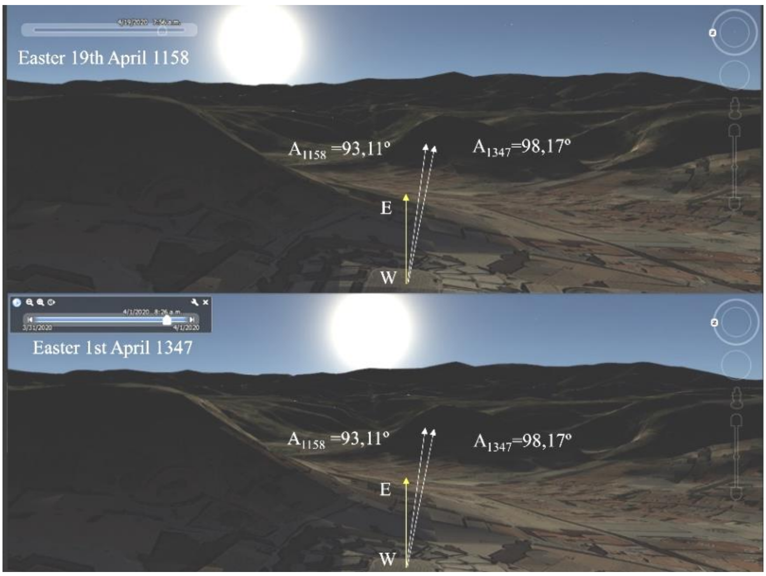

Fig. 14. Paschal calendar ctiterion; a) Easter date 19 th April 1158; b) Easter date 1 st April 1347.

- Topographic criteria. It consists in assuming that the orientation of the church is influenced by the surroundings of a characteristic landscape. In Antoon van den Wyngaerde's View of the City of Tortosa (1563), the Romanesque façade appears and the Coll de l'Alba (330 m) (AColl Alba = 86.92º), the old Via Augusta and the profile of the cusp of the Creu mountain (383m), (ACreu = 97.76º) where the cross that dominates the Territory of Tortosa is located. This orientation is very similar to that of the current cathedral (A1347 = 98.17º). It is a symbolic point, since in the cathedral ceremony there was the custom of blessing the city and the end of Tortosa from both crosses (Fig. 15).

- Constructive criterion of considering the pre-existing constructions, either the mosque or the remains of the Roman Temple referred to by Despuig. The superposition of the 1901 excavations of Joan Abril i Guanyabens (1852-1939) with the current cartography (Ac.100 = 108.64º) (Fig. 16) is very similar to the orientation to the Kaaba Ac.717 = 108.25º and therefore at its verification range [104.50º-112.00º], although the remains found belong to Roman construction, and at the moment, no Andalusian archaeological remains have been found in this area.

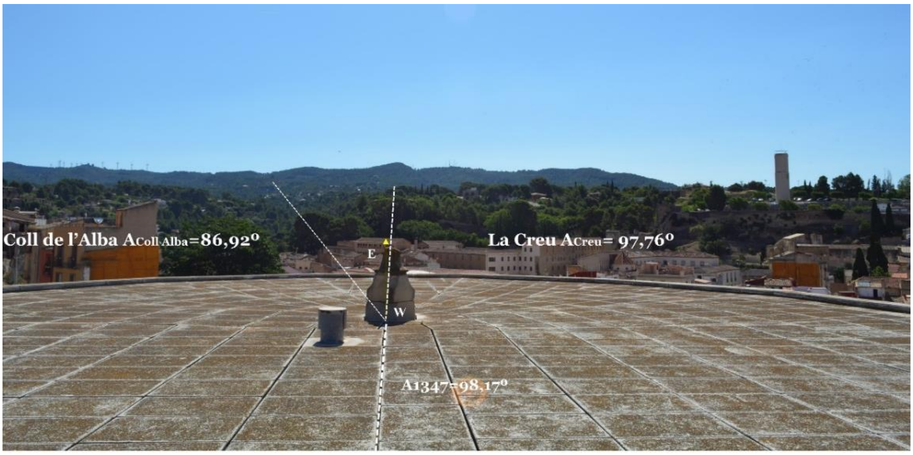

Fig. 15. Alignment of the cathedral crosses with the cusp of the Creu (383 m)

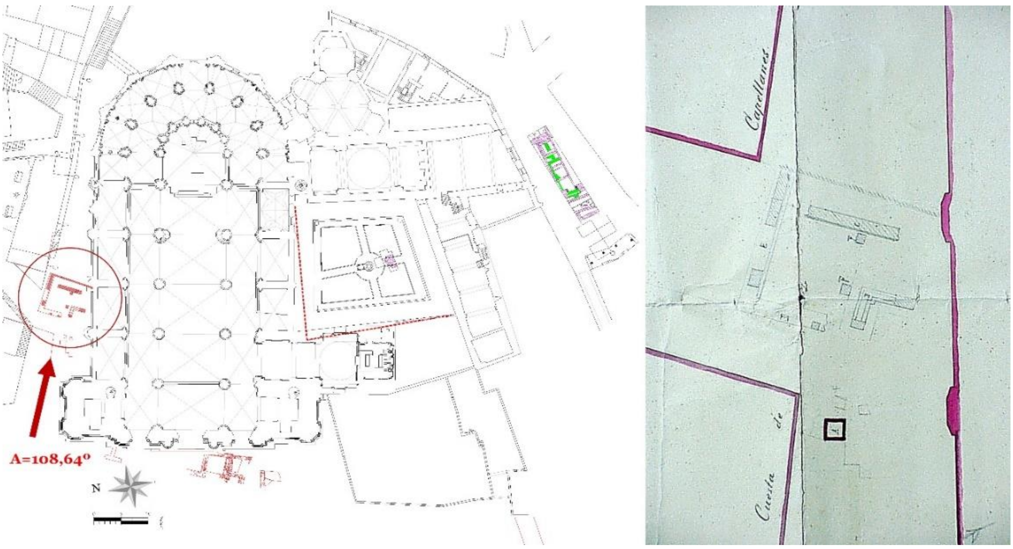

Fig. 16. Cathedral excavation alignment Joan Abril i Guanyabens (1901)

This analysis shows that the Romanesque cathedral causes a redirection on the existing archaeological remains of Roman tradition Ac.100 = 108.64º, and which could also be compatible with the hypothesis of the alignment of the Turtusha mosque Ac.717 = 108.25 °. The possible hypotheses of the Romanesque cathedral could have azimuths; A1158-I3 = 95.39º A1158-I4 = 93.11º and A1158-I6 = 98.00º

The A1158-I4 = 93.11º could represent a canonical orientation, it would be within a high precision of the interval [86.25º-93.75º], for an orientation [E-W = 90º] (Fig. 17).

The superposition of this hypothesis presents some distortions on the results of the geophysical survey of the cathedral's subsoil carried out in the campaign of (2012-13) [32] (Fig.

18), being more compatible with said survey A1158-I3 = 95.39º and A1158-I6 = 98.00º. Thus, the most probable value is the mean of the three azimuths (ea) which would result in A1158-Ii = 95.50º, mean square error of the mean (ec) = 2.00º.

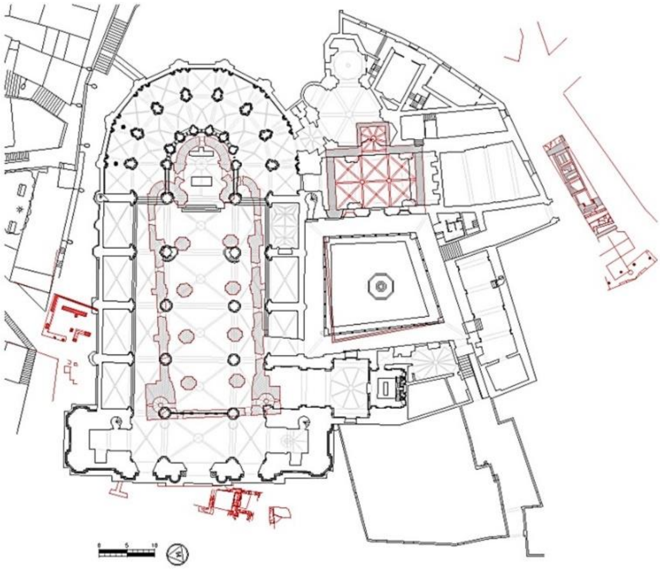

Fig. 17. Romanesque cathedral orientation superposition $A_{1158\text{-}I4}$ = 93.11º with the gothic cathedral A_{1347} = 98.17º

Orientation plotted analysis

The azimuth of the Romanesque cathedral, from the statistical point of view, would be in the interval [93.00º-97.99º] and, therefore, plotted with a precision [0.83% -2.22%] being its elevation lower within the interval [86.25º-93.75º] of the canonical value [EO = 90º] but also within the upper limit of a topographic orientation in the direction of ACreu = 97.76º. If the criterion was topographic, a simple two-pole alignment method was used, directing the line of sight to the top of La Creu mountain.

But if the canonical criterion was used, it was necessary to trace it with some method preserved in the monastic scriptories [33], either with instrumental systems, or by observing the solar star. Those that we have known today -until the twelfth century- are those defined by Vitruvius (c.80-20 BC) in De architectura, (M1, M2) [34], the methods transmitted by the gromatics (s. I-III), (M3, M4, M5, M6) [35], those of the Ripoll scriptorio (M7, M8) of Gisesmundus (c.800) [36] and Apocrypha of Gerbertus (c.1000) (M9, M10) [37], also raising the possible use of the compass (M11) [38].

The Vitruvian methods (Vitr, LI.VI.6) (c.80-20 BC), the (M1) determine a meridian orientation through two shadows and their execution requires six geometric operations [39]. The (M2) method (Vitr, LI.VI.12), traces an equinoctial orientation through three shadows and its execution requires up to fifteen geometric operations.

In the methods of Higynius Gromaticus (c.100) De limitibus constituendi, the (M3) (La. 166-208) a meridian orientation is determined by a procedure of observation of the prime hour. The method (M4) (La. 188), confirms a meridian orientation by observing the sun during the sixth hour. The method (M5) (La. 188-189) verifies a meridian orientation with two shadows, its execution requires six geometric operations. He (M6), (La. 189-191) executes an equinoctial orientation and its execution requires fifteen geometric operations [40].

In the methods of the Ars gromatica siue geometría Gisemundi (c.800) by Gisesmundus, the method (M7) determines an equinoctial orientation through two shadows and its execution requires four geometric operations. Method (M8) performs an equinoctial orientation through two shadows using a procedure that requires two geometric operations [41].

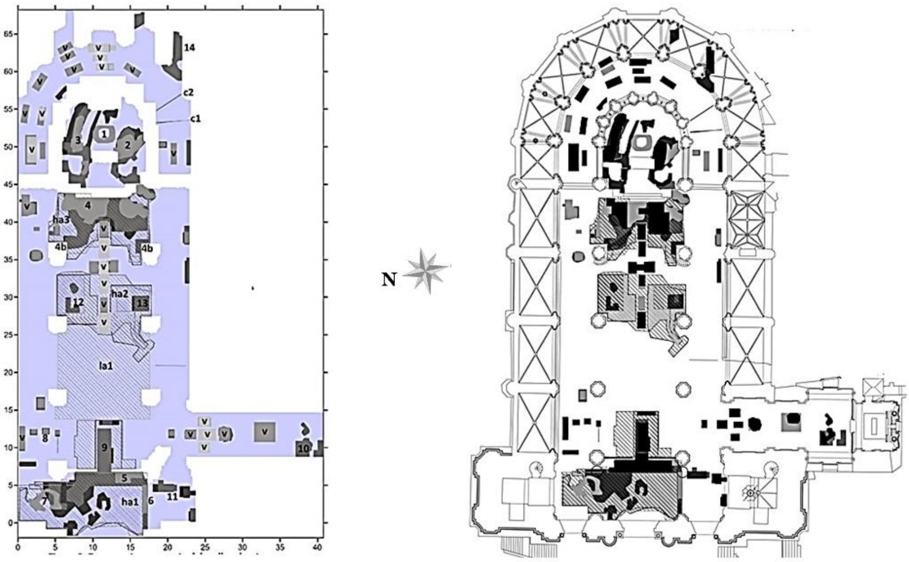

Fig. 18. Geophysical prospecting (2012-13)

In the apocryphal modes of Gerbertus [42] of the Geometria Incerti Auctoris, the (M9) is similar to the (M1) of gromatic, to the (M5) of Higynius Gromaticus and to the (M7) of Gisesmundus. The method (M10) is similar to the (M2) of Vitruvius and the (M6) of Higynius Gromaticus. The method (M11) proceeds by using the compass (c. 1150) it determines a meridian orientation by means of a magnetized needle.

Accuracy of the layout methods of the Cathedral of Santa María Tortosa (1158)

The eleven methods described above are based on both solar observation and operating systems based on geometric systems, if we analyze the results of the with the maximum tolerance (em, max = 3.11º) with an error (0.83%) (Table 3).

Methods M3, M4, and M11 are for appreciation and, therefore, are not strictly geometric. They are not included in the error analysis, since it is an appreciation value and, therefore, difficult to calibrate.

The M3 and M4 systems are the simplest and most direct, but also the least accurate. Regarding the compass, the change in magnetic declination should be taken into account, referred to the cathedral's major key (λ) [0 ° 31 '21.340' '], (ϕ) [40 ° 48 '51.413 "]. At present this value tends to an annual translation towards the East of (0.16º) [43], so in the year 1158 the true North would have a declination [+ 12.42º] [44], so hypothesis (M11) is unlikely.

Instrumental paths with two shadows (M1, M5, M7 and M9), one pre-Meridian and one post-Meridian are projected on a circumference. Vitruvius takes the first shadow five hours before noon, in such a way that he determines the radius of the circumference, generating a circle of great diameter, so the gnomon must have a small magnitude. The other methods draw the circumference and wait for the shadow to move in and out of it. Once the two points have been found, other variations are presented, Vitruvius (M1) performs a decussation that will join the center of the circumference to determine the meridian, in the same way Higinius (M5), but from the midpoint of the line that results from joining both points, with a margin of error of (eM1.5, = 0.14).

Table 3. Tolerances A_{1158} = 93.26º of the Romanesque cathedral

Tolerances of the layout methods of the orientation						
Method	Author	Orientation type	Tracing	Characteristics	Geometric operations	Error (max) =0.83%
M1	Vitruvius	Meridian	Instrumental	2 umbra	6	0.14
M2	Vitruvius	Equinoctial	Instrumental	3 umbra	15	0.06
M3	Hyginius Gromaticus	Meridian	Observation	Prime hour	-	-
M4	Hyginius Gromaticus	Equinoctial	Observation	Sixth hour	-	-
M5	Hyginius Gromaticus	Meridiano	Instrumental	2 umbra	6	0.14
M6	Hyginius Gromaticus	Equinoctial	Instrumental	3 umbra	15	0.06
M7	Gisemundus	Equinoctial	Instrumental	2 umbra	4	0.21
M8	Gisemundus	Equinoctial	Instrumental	2 umbra	2	0.42
M9	Apocrypha of Gerbertus	Meridian	Instrumental	2 umbra	6	0.14
M10	Apocrypha of Gerbertus	Equinoctial	Instrumental	3 umbra	15	0.06
M11	Compass	Meridian	Observation	Compass	-	-

The simplest method is that of Gisemundus (M7) since it joins the two points determining the equinoctial axis, thus being able to have a greater margin of error in its tracing (eM7 = 0.21), much higher than the (M1, M5) therefore, it has more geometric accuracy. For its part, the M8, based on two very close shadows close to the sixth hour, is the simplest, since it is based only on joining the ends of two close shadows of equivalent lengths, a method that can be imprecise because they are both very close shadow points. The M2, M6 and M10 methods use three shadows to determine the orientation, using much more complex geometric procedures, so due to their complexity it would lead them to operate with a much lower tolerance and accuracy (eM2,6,10 = 0,06).

Instruments of layout of the cathedral of Santa María Tortosa (1158)

It is probable that the instruments referred to in the Etymologiarum (XIX.18) of San Isidoro de Sevilla (c. 556-636) have been used to draw these methods and broadcast on De Universo (XXI.11) [45] by Harbanus Maurus (c. 776-856). Thus, the M1, M5 and M9, after bisecting the angle and leaning on the pole, the equinoctial axis would be marked through a pertice aequlite ad perpendiculum using lead and square as in the Siensa de atermenar de Berand Boysset (CBM 327, CBM 327, fol. 218 r, fol 220 r) [46] (Fig. 19), obtaining a tolerance percentage of up to (eM1.5.7 = 0.14). In the same work it is illustrated how this same axis was traced with the pole and made an extendere linean by using poles (CBM 327 fol. 251 r), which could have been used in the methods, M7 (eM7 = 0.21). and M8, (eM8 = 0.42). So, as for the instrumental methods, it is observed that the simplest from the geometric point of view to determine the equinoctial line are the Gisesmundus methods M7 and M8. The M8 system operates with very close shadows, so its extension may lose accuracy when having to work with very close angular distortion.

Methods M1, M5, and M9 require six compass operations, although if the plot is made at an interval before the third hour (3 in the morning) or after the ninth hour (3 in the afternoon), the angle suggested by the solar projection allows a precise bisector, thus that the meridian can be traced very precisely. For their part, the M2, M6 and M10 methods are the most complex geometrically, requiring up to fifteen operations.The methods with the greatest tolerance to absorb possible staking errors are the M7 and M8 of Gisemundus, these ecclesiae ad orientem had to be staked, or else, by means of two shadows, pre and post meridian, and traced only with an extendere lineam; or aligned, close to the sixth hour, by extending extendere lineam y pertice aequlite ad perpendiculum, using a plumb line and a square. In this context, the M7 method is deeply practical, the five observed shadows figuratively trace the analema of the solar course. It starts from a geometric construction of only two basic principles: drawing a circumference and having a gnomon squared on the plane where the circumference has been located.

The architect draws the circumference first thing in the morning, with a diameter smaller than the one that the shadow casts at that time (shadow 1), this is the only geometric operation that he has to perform. Then you just have to observe, mark the point where the shadow coincides with the circumference (shadow 2) and wait for the path of the sun to approach the base of the gnomon (shadow 3). Subsequently, it marks the point where the end of the shadow again coincides with the circumference (shadow 4), and thus, joining the points (2-4), it already obtains the E-W orientation without the need to use the longest shadow projection (shade 5) (Fig. 19).

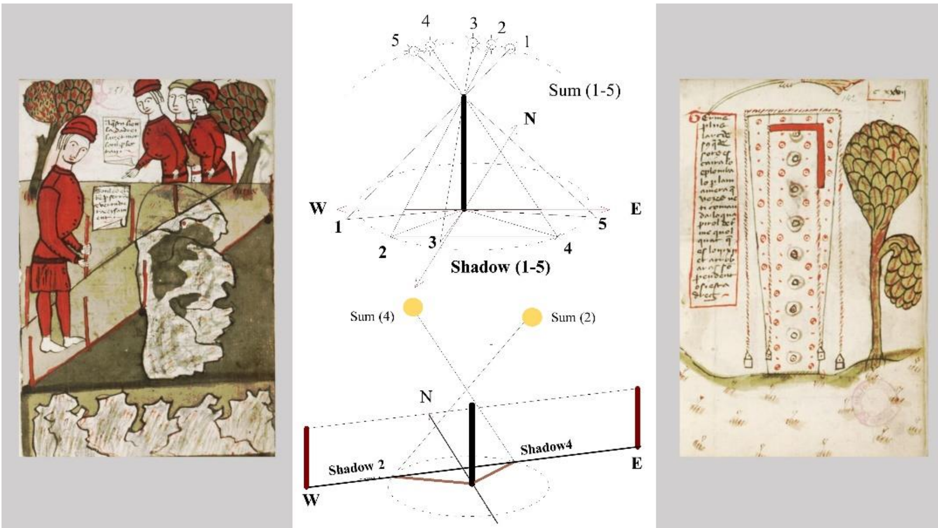

Fig. 19. Method M_8 of Gisemundus (c.800), operations *Siensa de atermenar de Berand Boysset* (CBM 327)

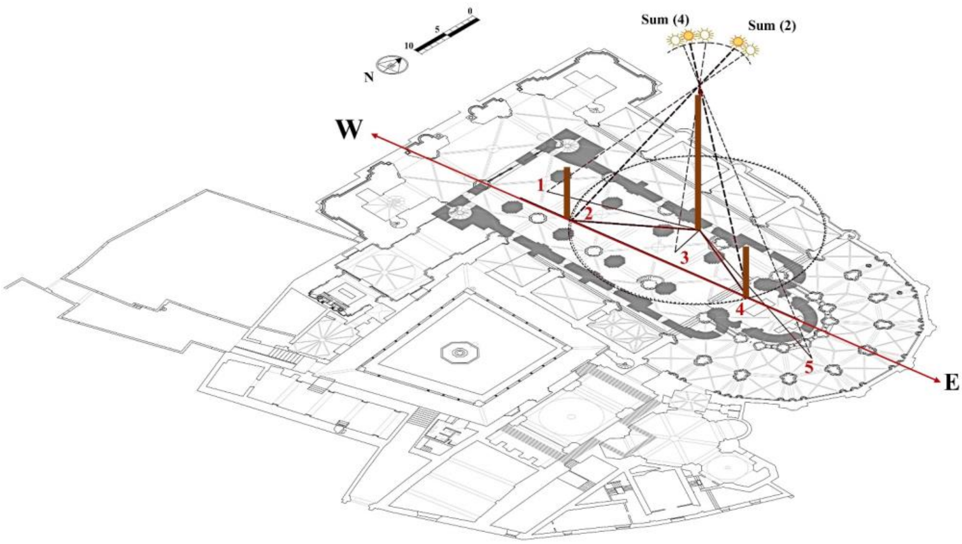

Fig. 20. The most probable orientation of the Romanesque cathedral Layout of the cathedral $A_{1158\text{-}Ii}$ with the gothic cathedral

Conclusion

The archaeological remains of the possible Roman Temple of Dertosa Ac.100 = 108.64º are compatible with the superposition of the Turtusha mosque and with the direction of the Kaaba in Mecca (Ac.717108.25º). The most probable orientation of the Romanesque cathedral would be A1158-Ii = 95.50º, with a range [93.00º-97.99º] (Fig. 20), whose interval would cover from the canonical orientation from East to West, to the topographic criterion to direct it to the top of the Creu (383 m) ACreu = 97.76º.

The Romanesque cathedral causes a redirection on the existing archaeological remains of Roman tradition, which is oriented with great accuracy Ecclesiae ad orientem, in accordance with liturgical treatises. The Gothic cathedral A1347 = 98.17º is reconsidered on the Romanesque one without being able to determine its axis, since it is built as a bypass on the ecclessia vetulam, and, therefore, the difference between the two is justifiable. Both orientations in turn preserve the visual horizon of the Coll del Alba mountains (330 m) in the old Augusta road AColl Alba = 86.92º.

We cannot establish which of the methods and instruments was used to determine the orientation of the Romanesque cathedral, but we have shown that the Ars gromatica siue Gisemundi geometry (scripta c. 800) is the one that can trace an orientation with the least error. The codicological origins of Gisesmundus are located in the Pseudoboethii (scriptum VIII century), of which two copies are preserved, Codex Parisinus BN 8812 (c. 800-833) of southern France (110/097) and the Codex Riuipullensis 106 (c. 850-900) from the Ripoll monastery (118/096). There could be speculation of the connection with the Ripoll monastery and desk through the abbot of the Ripoll monastery and San Juan de las Abadesas, Ponç de Monells (C.1120-1193) who was bishop of Tortosa (1165-1193), before the evidence of the Geometria incerti auctoris of Gerberto of the codex 80 (ACTo) of the second half of the XII century.

Abbreviations

ACTo. Archivo Capitular de Tortosa
CBM. Carpentras Bibliothèque Municipale
(De Uni.) MPL111, 1864. De Universo Rabanus Maurus.
(Etym.) Lindsay. 1911.. Etymlogiarum sive Originvm libri XX.
(*La.*) Lachman. 1848. Die Schriften der Römischen Feldmesser Herausgegeben und erläutert von F. Blume (et alt).
(Vitr.) Rose 1899. Vitruvii. De architectura Libri Decem. Valentinus Rose.

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