APPLICATION OF RIPLEY’S K-FUNCTION IN RESEARCH ON PROTECTION OF UNDERGROUND INFRASTRUCTURE AGAINST SELECTED EFFECTS OF SUFFOSION

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Abstract

Intense leakages from damaged water mains wash out soil particles from the solid matrix creating the threat to the stability of the existing infrastructure. One of the proposals for protecting the infrastructure is the idea of a protection zone, defined as an area around buried water supply pipes, where outflow of water on the soil surface (creation of suffusion holes) is possible after a presumptive failure of the pipe and where the enterprise managing a water distribution system would be responsible for land development. Determining a protection zone should be preceded by spatial characteristics of the pattern of suffusion holes to indicate a method for subsequent analyzes. The aim of this paper was using Ripley’s K-function to analyze spatial patterns of suffusion holes created on the soil surface by water leaking from a buried water pipe after a network breakage. The analyses were conducted for 2 values of leak area and 3 values of soil compaction (6 different cases) on the basis of measurements obtained during laboratory simulation of buried water pipe breakage. The results of investigations indicated that suffusion holes resulting from water outflow on the soil surface are randomly located for all but one analyzed case.

Keywords: Ripley’s K-function, Infrastructure protection, Pressure pipes failure, Water outflow

Introduction

Leakages from water mains caused by pipe breakages are often connected with rapid flow of great amount of water through soil or ground [1]. If the soil is internally unstable, fine particles are washed out from the solid matrix and transported through the pores causing the phenomenon of suffosion. As a result, water flowing with soil particles at first creates empty spaces below soil surface and then depression, hollows or holes in the soil surface, called here suffusion holes.

The phenomenon of suffosion is particularly dangerous in urbanized areas, because these areas are characterized by a higher population density than non-urbanized areas, more compact and often higher built-up areas, as well as the presence of industrial infrastructure. Empty spaces below the soil surface create the threat to the stability of objects, especially building foundations and underground pipelines. The risk of the phenomenon of suffosion in urbanized areas is most often associated with the presence of water supply systems, because their operation is accompanied by failures associated with outflows of water into the ground. Moreover, water mains are usually located along roads, where density of infrastructure is often the highest. Thus, different kinds of activities are undertaken to limit the problem of leakages from water pipes – methods for leak detection or location are still improved [2-7], high-tech
methods are used for pipes condition assessment [8-10], attempts are being made to predict water pipes failures [11-14]. The mentioned activities are very important, but not sufficient, because water pipes failures and breakages occur even in well-managed and operated water supply systems throughout their life cycle and are problems for water companies in both developing and developed countries all over the world. Moreover, water pipes failures and breakages are often random, caused or influenced by different factors, sometimes impossible to foresee [15,16]. In addition, the technical and financial limitations of many water companies cause that the latest methods of leak detection and control are not available to everyone, and the renovation or replacement of technical unsatisfactory pipes cannot be carried out immediately. It can therefore be assumed that undesirable outflows of water from distribution networks caused by failures will be a current problem for many years, so it is justified to take all actions aimed at limiting their negative effects, including infrastructure protection.

One of the proposals for reducing adverse results of suffosion holes creation is the idea of a protection zone, defined as an area around buried water supply pipes, where outflow of water on the soil surface is possible after a presumptive failure of the pipe [17-20]. Decisions about developing a protection zone would be taken by companies managing water distribution systems so as to minimize damages of infrastructure within the zone in the case of a water network failure. Infrastructure and settlement in the zone should be carefully planned in order to exclude the possibility of diminishing stability of objects and to limit the social, economic and environmental costs in the case of leakage from a water pipe. The proposed method of protection zone is aimed at limiting the effects associated with soil suffosion after a possible failure, thus it is a different approach than the prevention of accidents mentioned above [2-14]. However, it should be emphasized that both approaches complement each other.

Determining a protection zone is a difficult task. When considering the problem of unsealing of an underground water pipe, three basic, interrelated processes can be distinguished: pressure water flow in a closed pipe, water outflow through the hole in the pipe to the ground, and water flow in a porous medium (soil). Thus, a place of water outflow on the soil surface is influenced by many factors connected with hydraulic condition of water flow both in a pipe and in the soil. Therefore, the determination of the protection zone must be preceded not only by labor-intensive and time-consuming laboratory and in-situ investigations, but also by thorough theoretical analysis of the results of experimental investigations regarding their utility to determine protection zones. The theoretical analysis should include spatial characteristics of the pattern of suffosion holes to indicate a method for determining a protection zone size. If spatial pattern of points corresponding to the suffosion holes occurs regular, an attempt can be undertaken to determine protection zone dimensions on the basis of classical mathematics rules. If each point in a study area occurs randomly located, the zone determination becomes complicated and it is necessary to use more advanced methods, such as chaos theory or fractal geometry.

One of the tools for analyzing data on the locations of events or objects is a Ripley’s K-function [21]. The events or objects are considered as geometric points on a plane (usually), along a line or in space, and are described by Cartesian or polar coordinates. The expected number of points within a local neighborhood at any point in a study area is compared to the expected density assuming complete spatial randomness (CSR) [22].

Recent applications of Ripley’s K-function are very broad. For example, it is used to access spatial patterns of trees [23-25], non-homogenous granular blend [26], neurons [27, 28], clustering of proteins in membrane microdomains [29] or proteins colocalization [30], to analyze geographical concentration of economic activities [31, 32] or to identify flood frequency [33]. The aim of my paper is using Ripley’s K-function to analyze spatial patterns of suffosion holes created on the soil surface by water leaking from a buried water pipe after a network breakage. The presented investigations are a part of the wide range research, focusing on determining the size of a protection zone around the area where a water pipe is particularly exposed to leakage [34-37].
Experimental part

Materials

Data for analysis were obtained during laboratory investigations of water outflow on the soil surface after a buried water pipe failure. The pipe supplied by water from a container located on the assumed height was intentionally damaged and water leaked through a gap in the whole circumference of the pipe. The laboratory investigations were conducted for two pipe diameters $d_i$ – 6mm (leak area $A_L = 2.83\,\text{cm}^2$) and 40mm ($A_L = 18.84\,\text{cm}^2$), and for three variants of soil compaction. Soil compaction was characterized by the index $I_s$ defined as a ratio of actual soil dry density to maximum dry density according to standard Proctor compaction test [38]. The investigations were conducted for $I_s$ equaled 0.75, 0.85 and 0.95 for each leak area. The experiment was repeated 7 times for each case. In a single repetition, from 1 to 5 suffosion holes were formed. A scheme of the laboratory setup reflecting real conditions scaled 1:10, as well as details about the used materials and realization of the experiments are given in the papers of Iwanek et al. [18, 39, 40].

During investigations, each suffosion hole creating on the soil surface of the laboratory setup was represented by the farthest point of the suffosion hole from the leak in the pipe. The point was located by coordinates $(x, y)$ (Fig. 1). The origin of the coordinate system is directly above the leak. The suffosion hole’s shape and size were not taken into account in this study, however they were a subject of other analyses [41].

Fig. 1. Location of a suffosion hole [42]:
1 – suffosion hole on the sand surface; 2- box filling by sand; 3 – buried water pipe;
4 – place on the sand surface straight above a leak in the pipe (the coordinate system origin);
5 – circle determining location of the farthest point of a suffosion hole from the leak

To evaluate spatial patterns of suffosion holes, a rectangular study area with the centre in the coordinate system origin was assumed. Dimensions of the area were determined as:

$$a = 2 \cdot \max\{|x_i|\},$$

$$b = 2 \cdot \max\{|y_i|\},$$

where: $a$ and $b$ – the study area dimension parallel to the $X$ axis and to the $Y$ axis, respectively; $x_i$ and $y_i$ – coordinates of the $i^{th}$ point ($i = 1, 2, \ldots, N; N$ – total number of observed points).

Methods

The analytical part of investigations included testing on the basis of Ripley’s $K$ function whether a given distribution of points within a defined study area is a realization of a homogeneous Poisson process (complete spatial randomness). Values of the function for a homogeneous Poisson point process are calculated as:

$$K(r) = \pi \cdot r^2,$$

where: $r$ – assumed distance from an arbitrary point.

The statistic that is the most commonly used to estimate the $K(r)$ function is:

$$\hat{K}(r) = A/N^2 \cdot \sum_{i=1}^{N} \sum_{j=1, j\neq i}^{N} [w_{ij}^{-1} \cdot I(d_{ij})],$$

where $w_{ij}$ is the edge correction term, and $I(d_{ij})$ is the indicator function.

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where: \( A \) – study area, \( N \) – observed number of points in the study area, \( w_{ij} \) – correction factor for the edge effects reduction, \( d_{ij} \) – distance between the \( i^{\text{th}} \) and \( j^{\text{th}} \) points, \( I(d_{ij}) \) – indicator function according to the formulae:

\[
I(d_{ij}) = 1 \text{ if } d_{ij} \leq r
\]

or

\[
I(d_{ij}) = 0 \text{ if } d_{ij} > r
\]

The edge correction factor \( w_{ij} = 1 \), if a circle with a radius of \( d_{ij} \) (centered on the \( i^{\text{th}} \) point and passing through the \( j^{\text{th}} \) point) was completely contained in the study area. Otherwise, \( w_{ij} \) equaled the proportion of the circumference of the circle that is inside the study area [43-45].

The estimator \( \hat{K}(r) \) determined on the basis of laboratory observations according to the formula (4), was compared to the theoretical value of \( K(r) \) for a homogeneous Poisson point process. For complete spatial randomness the equation \( \hat{K}(r) = K(r) \) is satisfied. If \( \hat{K}(r) > K(r) \) significantly, the points should be considered as aggregated at distance \( r \). An inequality \( \hat{K}(r) < K(r) \) indicates regularity of a point pattern. A statistical significance of the comparisons’ results was evaluated using the student’s \( t \)-test. Although calculation can be conducted for any \( r \), it is common practice to assume the maximal value of distance \( r \) as one-half the shortest dimension of the study area [45].

Results and discussion

The spatial patterns of points representing suffosion holes are given in figure 2. The number of points is different for each case of \( A_L \) and \( I_s \), because water seldom appeared in only one place on the surface during laboratory investigations and number of the places of water outflow was different in subsequent repetitions of the experiment [46].

Fig. 2. Spatial patterns of suffosion holes in experiments conducted for different values of \( A_L \) and \( I_s \)
The highest number of points (29) was obtained for the case of $A_L = 2.83\text{cm}^2$ and $I_s = 0.95$ and in this case the points are scattered over the largest area. The lowest number of points (14) was obtained for the case of $A_L = 18.84\text{cm}^2$ and $I_s = 0.95$, but the area covering the points was not the lowest for this case. It can be noticed that both for $A_L = 2.83\text{cm}^2$ and $A_L = 18.84\text{cm}^2$ the area over which the points are scattered is larger for higher $I_s$. The location of the points seems to be chaotic in all cases of $A_L$ and $I_s$, but the visual assessment of the patterns is not sufficient for reliable conclusions.

The results of testing distribution of points are presented in Figure 3. Clear discrepancies in fitting the estimated points to the graph of the function $K(r)$ are visible in one case only – for $A_L = 2.83\text{cm}^2$ and $I_s = 0.95$. In the rest cases the estimated points are close to the line $K(r)$ suggesting random distribution of the analysed points. The results of statistical calculations using the student’s $t$-test confirmed the above insights (Table 1). For all but one cases of $I_s$ for $A_L = 2.83\text{cm}^2$ and for all cases of $I_s$ for $A_L = 18.84\text{cm}^2$ the statistic $t$ ($t$-value) occurred outside the critical region. The case of $I_s = 0.95$ for $A_L = 2.83\text{cm}^2$ occurred the exception for which the statistic $t$ was inside the critical region. Thus, the results of calculations indicated that for the vast majority of analyzed cases there is no reason to reject the null hypothesis $\hat{K}(r) = K(r)$ with a significance level of 0.05 and it can be claimed that distributions of points corresponding to locations of water outflow on soil surface from a damaged buried pipe, within all analyzed study areas are characterized by total spatial randomness. Similar results were obtained during the previous investigations [42] conducted for 6 different values of hydraulic pressure head in a pipe – for all analyzed cases, locations of points of water outflow occurred random. The most likely cause of the random location of the suffusion holes is the complexity of the phenomenon of water outflow from the pressure pipe into the ground and the multitude of factors related to this phenomenon.

![Graphs of the function $K(r)$ for a homogeneous Poisson point process according to eq. (3) and estimator $\hat{K}(r)$ according to eq. (4) for 6 sets of points obtained for different values of $A_L$ and $I_s$](image)

**Table 1.** Evaluation of statistical significance of the hypothesis testing

<table>
<thead>
<tr>
<th>$A_L$ cm$^2$</th>
<th>$I_s$</th>
<th>Mean $\hat{K}(r)/K(r)$</th>
<th>Expected $\hat{K}(r)/K(r)$</th>
<th>$t$-value</th>
<th>Critical region</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.83</td>
<td>0.75</td>
<td>0.98</td>
<td>1</td>
<td>-0.1963</td>
<td>(-∞, -2.262&gt; or &lt;2.262, +∞)</td>
</tr>
<tr>
<td>2.83</td>
<td>0.85</td>
<td>0.90</td>
<td>1</td>
<td>-1.3853</td>
<td>(-∞, -2.093&gt; or &lt;2.093, +∞)</td>
</tr>
<tr>
<td>2.83</td>
<td>0.95</td>
<td>1.86</td>
<td>1</td>
<td>5.1880</td>
<td>(-∞, -2.080&gt; or &lt;2.080, +∞)</td>
</tr>
<tr>
<td>18.84</td>
<td>0.75</td>
<td>1.15</td>
<td>1</td>
<td>1.5125</td>
<td>(-∞, -2.145&gt; or &lt;2.145, +∞)</td>
</tr>
<tr>
<td>18.84</td>
<td>0.85</td>
<td>1.13</td>
<td>1</td>
<td>1.6804</td>
<td>(-∞, -2.145&gt; or &lt;2.145, +∞)</td>
</tr>
<tr>
<td>18.84</td>
<td>0.95</td>
<td>1.22</td>
<td>1</td>
<td>1.7631</td>
<td>(-∞, -2.069&gt; or &lt;2.069, +∞)</td>
</tr>
</tbody>
</table>
Conclusions

A Ripley’s $K$-function is a tool that can be used for analyzing the spatial patterns of suffosion holes resulting from water pipes failures. The analyses were conducted for 6 different cases of leak area (2 values) and soil compaction (3 values) on the basis of measurements obtained during laboratory simulation of buried water pipe breakage. The results of investigations indicated that suffosion holes resulting from water outflow on the soil surface are randomly located for all but one analyzed case. Taking into account the results of this paper as well as the previous investigations it can be supposed that spatial randomness is the feature of suffosion hole’s locations. This information is important regarding the possibility of determining a size of a protection zone near buried water supply pipes, designated to ensure the stability of nearby infrastructure. It gives direction to the further investigations – classical mathematics can be inapplicable in the case of randomly located points and research should be focused on more advanced methods, e.g. fractal geometry. However, investigations on suffosion holes distribution should be continued to formulate the final conclusions, especially in terms of other parameters (not only $A_L$ and $I_L$) influencing the process of water outflow on the soil surface after a failure of a buried water pipe.

All activities aimed at determining the protection zone are important in a practical aspect. Information about the size of the safety zone would help designers to plan both water supply routes and the location of other underground and above-ground infrastructure. Such information would also help water network operators in making decisions about actions to prevent or at least reduce the number of accidents connected with suffusion in urbanized areas.

References


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